Mechatronic Systems Design

MEC301

Asst. Prof. Dr.Ing.

Mohammed Nour A. Ahmed

mnahmed@eng.zu.edu.eg

https://mnourgwad.github.io

Lecture 3: **Modeling and Simulation**

Zagazig University | Faculty of Engineering | Computer and Systems Engineering Department | Zagazig, Egypt

Modeling

- *Modeling is the process of representing the behavior of a real system* by a collection of mathematical equations and logic.

- Models are cause-and-effect structures—they accept external information and process it with their logic and equations to produce one or more outputs.
  - Parameter is a fixed-value unit of information
  - Signal is a changing-unit of information

- Models can be text-based programming or block diagrams

Math Modelling Categories

- Static vs. dynamic
- Linear vs. nonlinear
- Time-invariant vs. time-variant
- SISO vs. MIMO
- Continuous vs. discrete
- Deterministic vs. stochastic

Static vs. Dynamic

- Models can be static or dynamic
  - **Static models** produce no motion, fluid flow, or any other changes.
  - Example: Battery connected to resistor $v = iR$
  - **Dynamic models** have energy transfer which results in power flow. This causes motion, or other phenomena that change in time.
  - Example: Battery connected to resistor, inductor, and capacitor

$$v = Ri + L\frac{di}{dt} + \int \frac{1}{C} idt$$

Linear vs. Nonlinear

- **Linear** models follow the **superposition principle**
  - The summation outputs from individual inputs will be equal to the output of the combined inputs

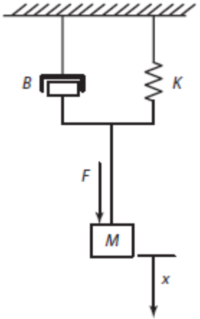
A system represented by $\mathcal{S}$ is said to be *linear* if for inputs $x(t)$ and $v(t)$, and any constants α and β, superposition holds—that is,

$$\begin{aligned} \mathcal{S}[\alpha x(t) + \beta v(t)] &= \mathcal{S}[\alpha x(t)] + \mathcal{S}[\beta v(t)] \\ &= \alpha \mathcal{S}[x(t)] + \beta \mathcal{S}[v(t)] \end{aligned}$$

- Most systems are nonlinear in nature, but linear models can be used to approximate the nonlinear models at certain point.

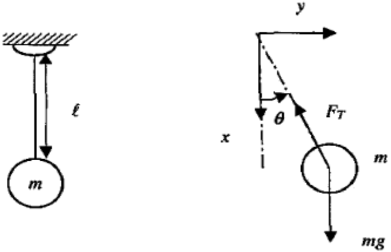
Linear vs. Nonlinear Models

- Linear Systems



$$\ddot{x}(t) = -\frac{B}{M}\dot{x}(t) - \frac{K}{M}(x(t) - x_0) + \frac{1}{M}F(t)$$

- Nonlinear Systems



$$-F_T\cos\theta + mg = m\left(-\ell\ddot{\theta}\sin\theta - \ell\dot{\theta}^2\sin\theta\right)$$

$$-F_T\sin\theta = m\left(\ell\ddot{\theta}\cos\theta - \ell\dot{\theta}^2\sin\theta\right)$$

Time-invariant vs. Time-variant

- The model parameters do not change in time-invariant models
- The model parameters change in time-variant models
  - Example: Mass in rockets vary with time as the fuel is consumed.

If the system parameters change with time, the system is time varying.

Time-invariant vs. variant

- Time-invariant

$$F(t) = ma(t) - g$$

Where m is the mass, a is the acceleration, and g is the gravity

- Time-variant

$$F = m(t)\, a(t) - g$$

Here, the mass varies with time. Therefore the ***model is time-varying***

Linear Time-Invariant (LTI)

- **LTI** models are of great use in representing systems in many engineering applications.
  - The **appeal is its simplicity and mathematical structure**.
- Although most actual systems are nonlinear and time varying
  - Linear models are used to approximate around an operating point the nonlinear behavior
  - Time-invariant models are used to approximate in short segments the system's time-varying behavior.

SISO vs. MIMO

- **Single-Input Single-Output** (SISO) models are somewhat easy to use. Transfer functions can be used to relate input to output.

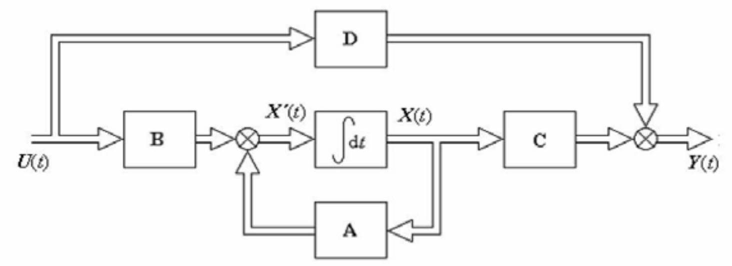
- **Multiple-Input Multiple-Output** (MIMO) models involve combinations of inputs and outputs and are difficult to represent using transfer functions. ***MIMO models use State-Space equations***

System States

- **Transfer functions**
  - Concentrates on the input-output relationship only.
  - Relates output-input to one-output only **SISO**
  - It hides the details of the inner workings.

- **State-Space Models**
  - ***States*** are introduced to get better insight into the systems' behavior. These states are a collection of variables that summarize the present and past of a system
  - Models can be used for **MIMO** models

SISO vs. MIMO Systems



Continuous vs. discrete

- **Continuous models** have continuous-time as the dependent variable and therefore inputs-outputs take all possible values in a range

- **Discrete models** have discrete-time as the dependent variable and therefore inputs-outputs take on values at specified times only in a range

Continuous vs. discrete

- **Continuous Models**

- Differential equations
- Integration
- Laplace transforms

- **Discrete Models**

- Difference equations
- Summation
- Z-transforms

Deterministic vs. Stochastic

- **Deterministic models** are uniquely described by mathematical equations. Therefore, all past, present, and future values of the outputs are known precisely

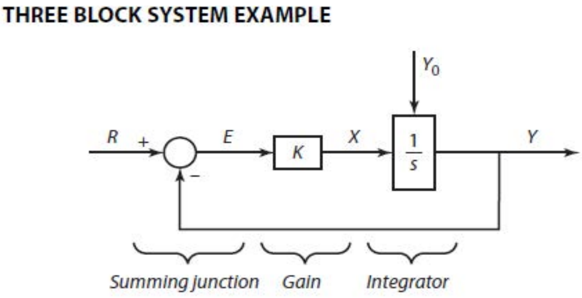
- **Stochastic models** cannot be described mathematically with a high degree of accuracy. These models are based on the theory of probability

Block Diagrams

- Block diagram models consist of two fundamental objects: *signal blocks and wires*.
  - A *block is a processing element which operates on input* signals and parameters to produce output signals
  - *A* wire is to transmits a signal from its origination point (usually a block) to its termination point (usually another block).

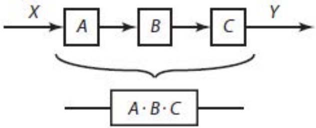
- Block diagrams are suitable to represent multi-disciplinary models that represent a physical phenomenon.

Block Diagram Example

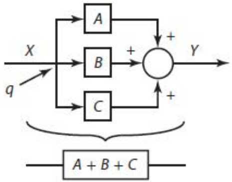


Block Diagrams Manipulation

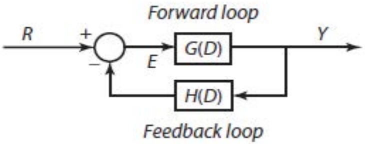
SERIES MANIPULATION—SERIES BLOCKS MULTIPLY



PARALLEL MANIPULATION—PARALLEL BLOCKS ADD



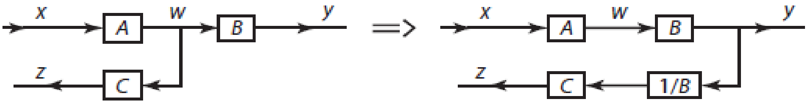
BASIC FEEDBACK SYSTEM (BFS) BLOCK DIAGRAM



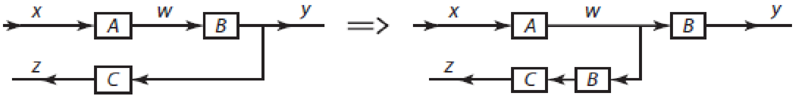
$$\frac{Y}{R} = \frac{G(D)}{1 + G(D) \cdot H(D)}$$

Block Diagrams Manipulation

PICK-OFF POINT SHIFTED DOWNSTREAM



PICK-OFF POINT SHIFTED UPSTREAM

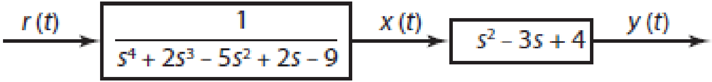


Block Diagrams: Direct Method Example

Consider the transfer function:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{s^2 - 3s + 4}{s^4 + 2s^3 - 5s^2 + 2s - 9}$$

We can introduce s state variable, x(t), in order to separate the polynomials

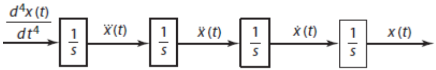


State Equation

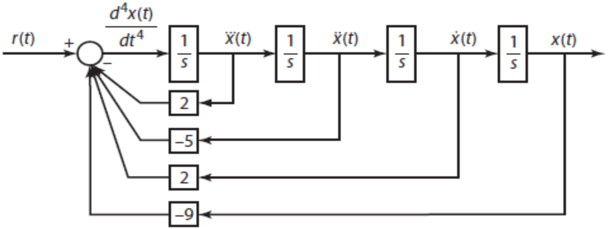
The differential equation is:

$$\frac{d^4x(t)}{dt^4} + 2\frac{d^3x(t)}{dt^3} - 5\frac{d^2x(t)}{dt^2} + 2\frac{dx(t)}{dt} - 9x(t) = r(t)$$

Put the needed *integrator blocks*:



Add the required *multipliers* to obtain the state equation:

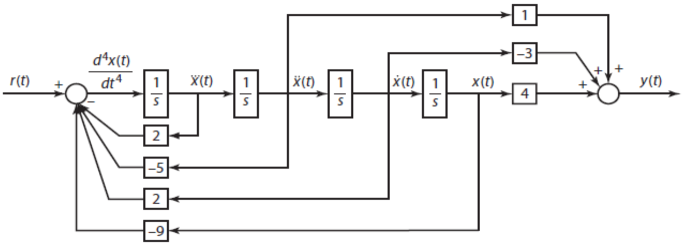


Output Equation

Repeat the same procedure for the output equation:

$$\ddot{x}(t) - 3\dot{x}(t) + 4x(t) = y(t)$$

Connect the two sub-blocks



Block Diagram Modeling: Analogy Approach

- Physical laws are used to predict the behavior (both static and dynamic) of systems.
  - Electrical engineering relies on Ohm's and Kirchoff's laws
  - Mechanical engineering on Newton's law
  - Electromagnetics on Faradays and Lenz's laws
  - Fluids on continuity and Bernoulli's law
- Based on electrical analogies, we can derive the *fundamental equations of systems* in five disciplines of engineering:
  - Electrical, Mechanical, Electromagnetic, Fluid, and Thermal.
- By using this *analogy* method to first derive the fundamental relationships in a system, the equations then can be represented in block diagram form, allowing secondary and nonlinear effects to be added.
  - This two-step approach is especially useful when modeling large coupled systems using block diagrams.

Power and Energy Variables: Effort & Flow

TABLE 9.1 Power and Energy Variables for Mechanical Systems

Energy Domain	Effort, e	Flow, f	Power, P
General	e	f	$e \cdot f$ [W]
Translational	Force, F [N]	Velocity, V [m/sec]	$F \cdot V$ [N m/sec, W]
Rotational	Torque, T or τ [N m]	Angular velocity, ω [rad/sec]	$T \cdot \omega$ [N m/sec, W]
Electrical	Voltage, v [V]	Current, i [A]	$v \cdot i$ [W]
Hydraulic	Pressure, P [Pa]	Volumetric flowrate, Q [m^3/sec]	$P \cdot Q$ [W]

Thanks for your attention.

Questions?

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mnahmed@eng.zu.edu.eg

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Robotics Research Interest Group (zuR^{2}IG)
Zagazig University | Faculty of Engineering | Computer and Systems Engineering Department | Zagazig, Egypt