Robotics

CSE4316

Assoc. Prof. Dr.Ing.
Mohammed Nour
mnahmed@eng.zu.edu.eg

https://mnourgwad.github.io/CSE4316

Lecture 6: Forward Kinematics

Lecture: 6

Forward Kinematics

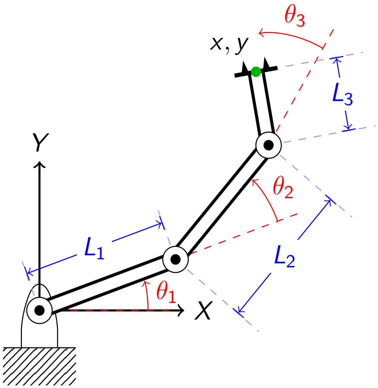
- Forward Kinematics
- Algebraic Approach
- Denavit–Hartenberg (DH) Convention

Forward Kinematics

- You have a robotic arm that starts out aligned with the x_0-axis.
- for a specific values of joint space $\mathbf{q} = [q_1, q_2, \cdots, q_n]^T$
- The Quest: What is the position of the robot arm tip?

To find the robot forward kinematics:

1. **Geometric Approach**
   - suitable for the simple situations.
   - For robots with more links and whose arm extends into 3D, the geometry gets much more tedious.
2. **Algebraic Approach**
   - Involves coordinate transformations.

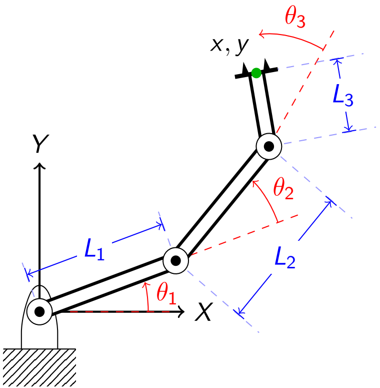


Forward Kinematics

Example

For the 3R arm, link lengths are L_1, L_2, L_3. For specific $\mathbf{q} = [\theta_1, \theta_2, \theta_3]^T$, get its tooltip position (the green dot in the $X_0 Y_0$ frame).

- **Geometric Approach**
  - just extend the results we already obtained with the 2R arm.
- **Algebraic Approach**
  - this approach we will learn next



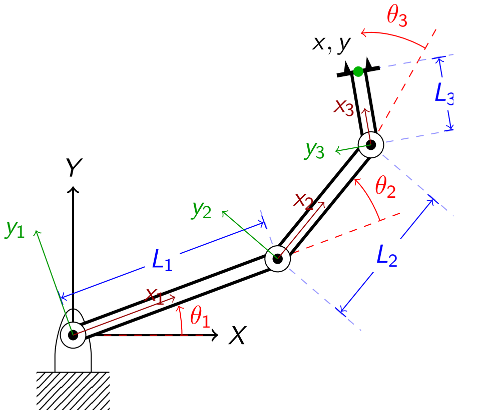
Forward Kinematics

Algebraic Approach

- we start by frame assignment following a **convention**
- we can get the position of the tooltip by:
  - rotating by θ_1 will put you in frame $\{1\}$.
  - Translate along the X_1 axis by L_1.
  - Rotating by θ_2 will reach frame $\{2\}$.
  - and so on until we are in frame $\{3\}$.

  $$H = R_z(\theta_1) * T_{x1}(L_1) * R_z(\theta_2) * T_{x2}(L_2) * R_z(\theta_3)$$

- tooltip position relative to frame $\{3\}$ is $(L_3, 0)$.
  - Multiplying H by that position vector will give the tooltip coordinates relative to frame $\{0\}$.



- We will develop a set of **conventions** that provide a **systematic procedure** for performing this analysis.
- It is possible to carry out forward kinematics analysis even **without** respecting these conventions,
  - as we did for the two-link planar manipulator example.
- However, the kinematic analysis of an n–link manipulator can be extremely **complex**
- the conventions introduced **simplify** the analysis considerably.
  - they give rise to a universal language with which robot engineers can communicate.
- A commonly used **convention for selecting frames** of reference in robotic applications is the **Denavit–Hartenberg** (DH)

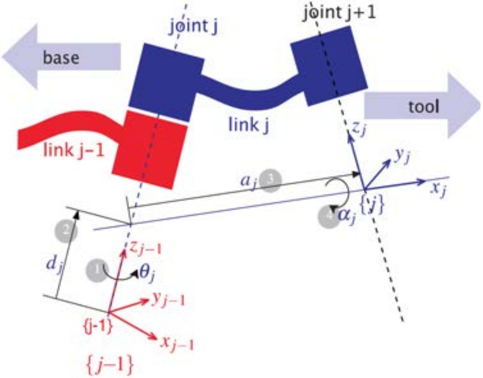
Denavit-Hartenberg (DH) Notation

- Each joint is assigned a coordinate frame. Using **DH convention**,

- In this convention, each homogeneous transformation A_i is represented as a product of four basic transformations

$$A_i = Rot_z(\theta_i) Trans_z(d_i) Trans_x(a_i) Rot_x(\alpha_i)$$

- the four quantities θ_i, a_i, d_i, α_i are parameters associated with link i and joint i
- They are called: link length, link twist, link offset, and joint angle, respectively



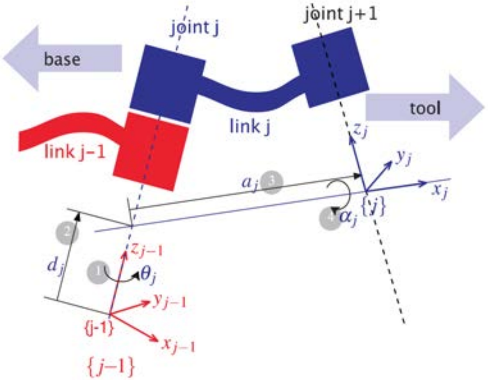
Peter Corke, *Robotics, Vision and Control*, Springer-Verlag, 2011

Denavit–Hartenberg Convention

DH Coordinate Frame Assumptions

- The axis x_i is perpendicular to the axis z_{i-1}.
- The axis x_i intersects the axis z_{i-1}.

DH Convention can NOT be applied if these assumptions are not fulfilled



Peter Corke, *Robotics, Vision and Control*, Springer-Verlag, 2011

Denavit–Hartenberg Convention

Summary of DH Convention Procedure

- Step 1: Locate and label the joint axes $z_0, \cdots, z_{n-1}$.
  - zi to be the axis of actuation for joint i + 1
  - z0 is the axis of actuation for joint 1, z1 is the axis of actuation for joint 2, for revolute joint, z_i is axis of revolution, for prismatic joint, z_i is its translation axis.
- **Step 2**: Establish the base frame. Set the origin anywhere on the z_0-axis. The x_0 and y_0 axes are chosen conveniently to form a right–handed frame.

For $i = 1, \cdots, n-1$, perform Steps 3 to 5.

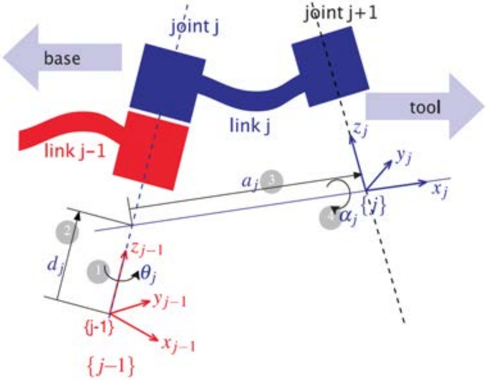
- **Step 3:** Locate the origin O_i where the common normal to z_i and z_{i-1} intersects z_i.
  - If z_i intersects z_{i-1} locate O_i at this intersection.
  - If z_i and z_{i-1} are parallel, locate O_i in any convenient position along z_i.
- **Step 4:** Establish x_i along the common normal between z_{i-1} and z_i through O_i, or in the direction normal to the $z_{i-1} - z_i$ plane if they intersect.
- **Step 5:** Establish y_i to complete a right–handed frame.

Denavit–Hartenberg Convention

Summary of DH Convention Procedure

- Step 6: Create a table of link parameters:

link	a_i	α_i	d_i	θ_i
1				
$\cdots$				



Joint angle	θ_i	angle from X_{i-1} to X_i about Z_{i-1}	revolute var.
Link offset	d_i	distance from O_{i-1} to X_i along Z_{i-1}	prismatic var.
Link length	a_i	distance between Z_{i-1} and Z_i along X_i	constant
Link twist	α_i	angle from Z_{i-1} to Z_i about X_i	constant

- Step 8: Form the homogeneous transformation matrices A_i by substituting the above parameters into A_i matrices:

$$A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Step 9:** Form the transformation matrix $T_n^0 = A_1 \cdots A_n$

This then gives the position and orientation of the tool frame expressed in base coordinates.

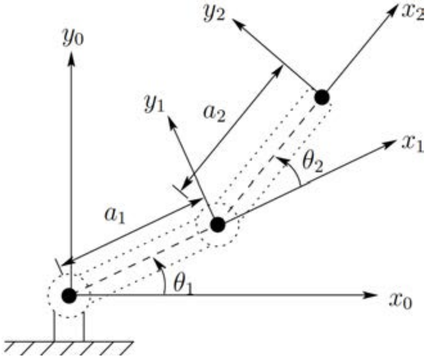
link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	0	0	θ_2^*

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = a_1\, c_1 + a_2\, c_{12} \quad y = a_1\, s_1 + a_2\, s_{12}$$



DH Convention Examples

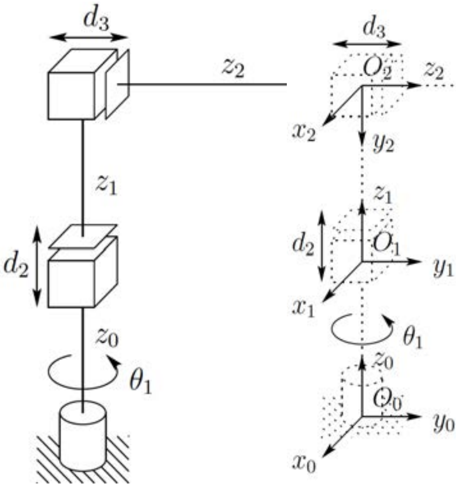
Three–Link Cylindrical Robot

link	a_i	α_i	d_i	θ_i
1	0	0	d_1^*	θ_1^*
2	0	-90	d_2^*	0
3	0	0	d_3^*	0

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



DH Convention Examples

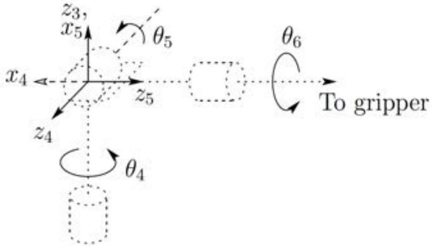
Spherical Wrist

link	a_i	α_i	d_i	θ_i
4	0	-90	0	θ_4^*
5	0	90	0	θ_5^*
6	0	0	d_6^*	θ_6^*

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^3 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Thanks for your attention.

Questions?

Assoc. Prof. Dr.Ing.
Mohammed Nour
mnahmed@eng.zu.edu.eg
https://mnourgwad.github.io/CSE4316

ZU Robotics Research Interest Group (zuR^2IG)
Robotics Research Interest Group (zuR^2IG)
Zagazig University | Faculty of Engineering | Computer and Systems Engineering Department | Zagazig, Egypt