Robotics

CSE4316

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Lecture 4: **Robot Kinematics (cont.)**

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Lecture: 4

Robot Kinematics (cont.)

- Robot Workspace, Reference Frames.
- Robot Kinematics
- Frames, Points and Vectors
- Basic Transformations
  - Translation along X–Axis
  - Translation along XY–Axis

Workspace

Workspace (Work Envelope)

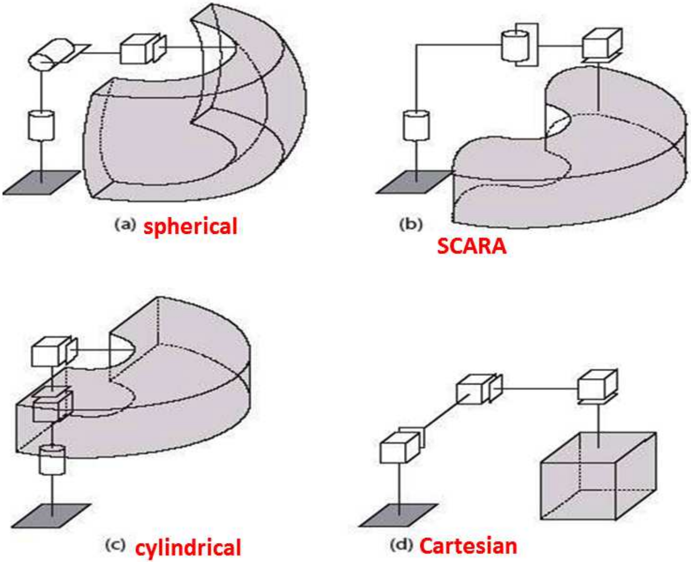
The Workspace of the manipulator is the total volume swept out by the end effector as the manipulator executes all possible motions.

Workspace is constrained by:

- Geometry of the manipulator.
- Mechanical constraint of the joints (a revolute joint may be limited to less than 360 degrees)

- **Reachable** Workspace: the entire set of points reachable by the manipulator.
- **Dextrous** Workspace: consists of those points that the manipulator can reach with an arbitrary orientation of the end effectors.
  - Dextrous Workspace is a subset of Reachable Workspace.

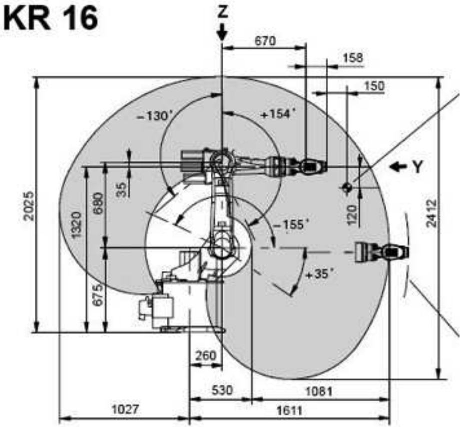
Workspace



Robot Characteristics and Performance Measure

- **Payload**: the max load a robot can handle
- **Reach**: The maximum distance a robot can reach within its work envelope.
- **Precision** (Accuracy, validity): how accurately a specified point can be reached.
- **Repeatability** (variability): how accurately the same position can be reached if the motion is repeated many times.

Robot Characteristics and Performance Measure

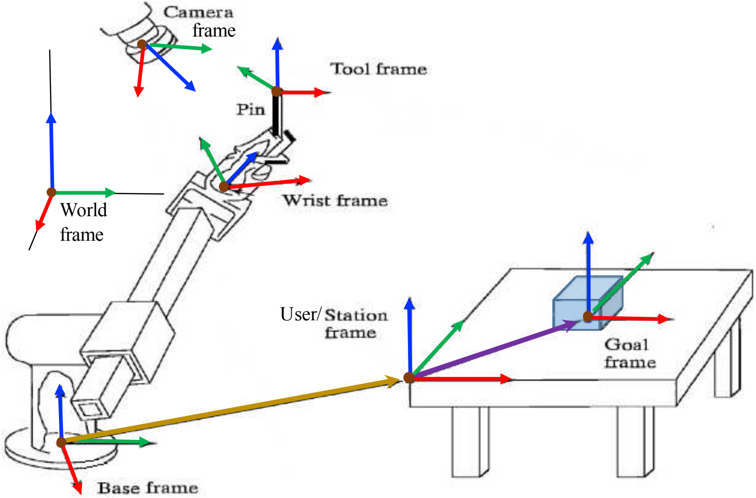


- The work envelope of the robot, as viewed from the side.
- payload at the wrist is 30 kg and repeatability is ±0.15 mm.

Reference Frames

Frames with Standard Names

Most robots may be programmed to **move relative** to either of these **reference frames**.



Reference Frames

Frames with Standard Names

World Reference Frame

A universal coordinate frame, as defined by the $x - y - z$ axes. In this case the joints of the robot move simultaneously so as to create motions along the three major axes.

Joint Reference Frame

used to specify movements of each individual joint of the Robot. In this case each joint may be accessed individually and thus only one joint moves at a time.

Tool Reference Frame

specifies the movements of the robot tool–tip relative to the frame attached to the hand. All joints of the robot move simultaneously to create coordinated motions about the Tool frame.

Robot Kinematics

Robot Kinematics

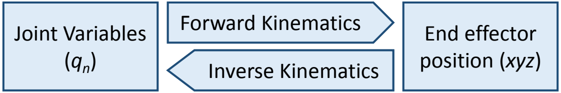
Kinematics

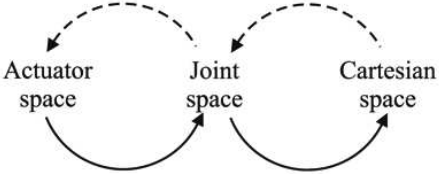
The branch of classical mechanics that describes the motion of objects without consideration of the forces that cause it

- The study of movement
- Why do you need it?
  - Determine endpoint position and/or joint positions
  - Calculate mechanism velocities, accelerations, etc.
  - Calculate force-torque
- Allows you to move between Joint Space and Cartesian Space

Robot Kinematics

- **Forward kinematics**: determines where the robot hand is (all joint variables are known)
  - you are given: length of each link, angle of each joint.
  - you find: position of any point (i.e. its (x, y, z) coordinates)



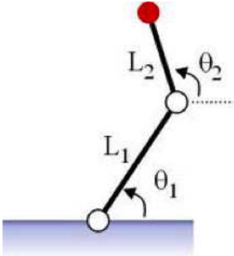


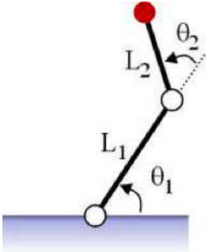
- **Inverse Kinematics**: to calculate what each joint variable is (If we desire that the hand be located at a particular point)
  - you are given: length of each link, position of some point on the robot
  - you find: The angles of each joint needed to obtain that position

Robot Kinematics

Joint Variables

- In forward kinematics we convert from **Joint space** to **Cartesian Space**
- To reach a specific point in 3D space we need to get the required joint angles





- **Absolute** (Sometimes)

$$x = L_1 \cos(\theta_1) + L_2 \cos(\theta_2)$$
$$y = L_1 \sin(\theta_1) + L_2 \sin(\theta_2)$$

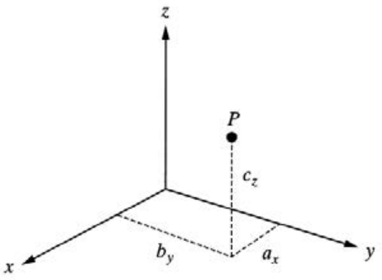
- **Relative** (usually)

$$x = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2)$$
$$y = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2)$$

Description of position and orientation

Frames, Points and Vectors

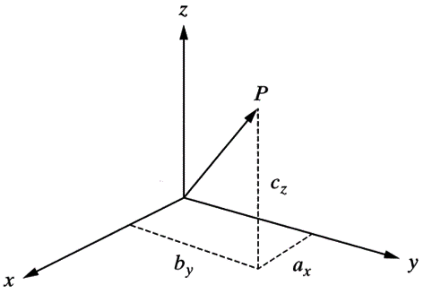
A **point** corresponds a particular location in the space.



3 coordinates relative to a reference frame:

$$P = a_x\hat{i} + b_y\hat{j} + c_z\hat{k}$$

A **vector** is defined by direction and magnitude.



3 coordinates of its tail and of its head:

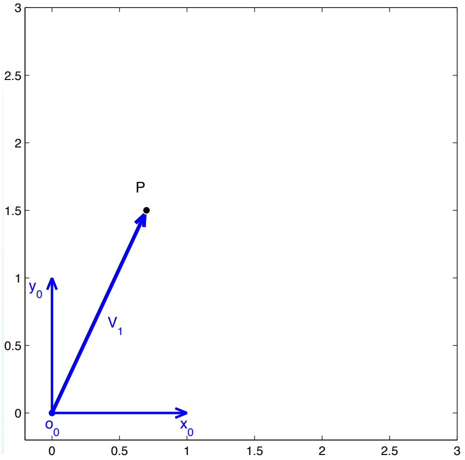
$$\vec{P} = a_x\hat{i} + b_y\hat{j} + c_z\hat{k}$$

Description of position and orientation

Frames

A coordinate frame in $\mathbf{R}^2$:

- the point $P = [x_0^p, y_0^p]^T$ can be associated with the vector $\vec{V}_1$.
- Here x_0^p denotes x-coordinate of point P in $\{x_0, o_0, y_0\}$-frame (i.e. in the 0-frame).

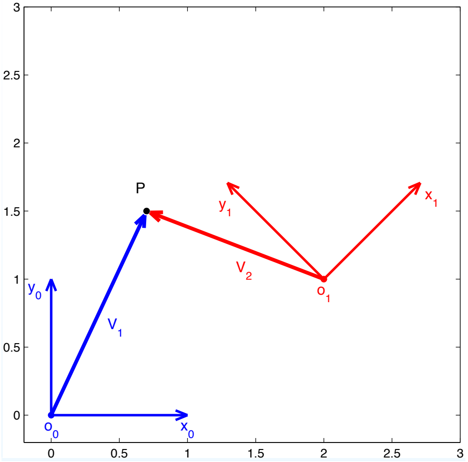


Description of position and orientation

Frames

Two coordinate frames in $\mathbf{R}^2$:

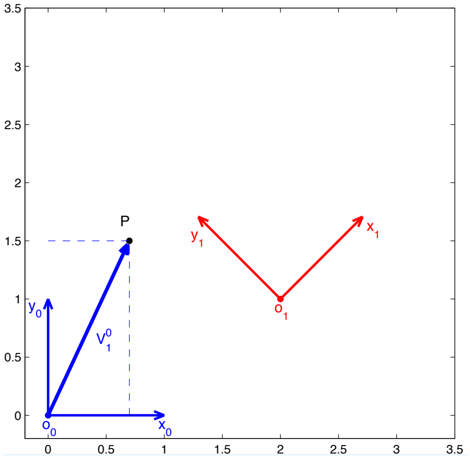
- the point $P = [x_0^p, y_0^p]^T = [x_1^p, y_1^p]^T$ can be associated with vectors $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$.
- Here x_1^p denotes x-coordinate of point P in $\{x_1, o_1, y_1\}$-frame (i.e. in the 1-frame).



Description of position and orientation

Frames

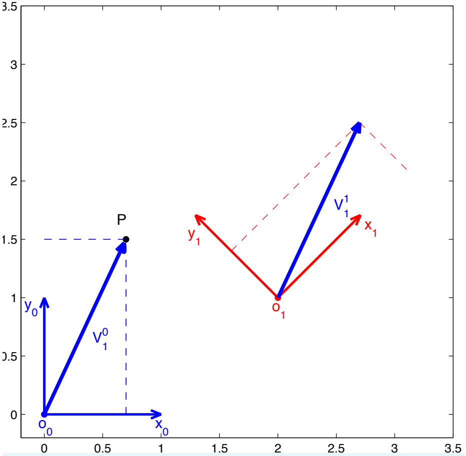
- The coordinates of the vector $\vec{\mathbf{V}}_1$ in the 0-frame $[x_0^p, y_0^p]^T$
- What would be coordinates of $\vec{\mathbf{V}}_1$ in the 1-frame?



Description of position and orientation

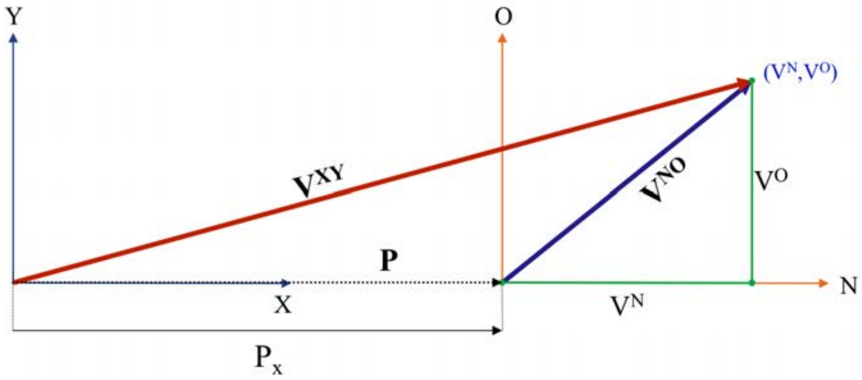
Frames

- We need to consider the vector $\vec{\mathbf{V}}_1^1$ of the same direction and magnitude as $\vec{\mathbf{V}}_1^0$ but with the origin in o1.
- Conclusion: we can sum vectors only if they are expressed in parallel frames.



Basic Transformations

Translation along X–Axis

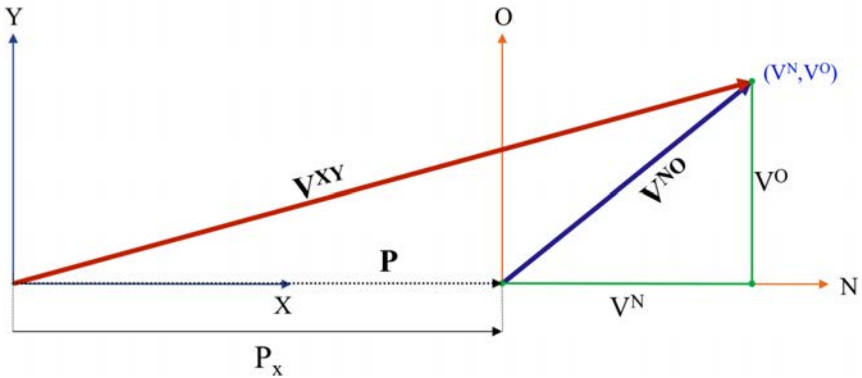


p_x = distance between the XY and NO coordinate frames

$$v^{XY} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}, \quad v^{NO} = \begin{bmatrix} v_N \\ v_O \end{bmatrix}, \quad P = \begin{bmatrix} p_x \\ 0 \end{bmatrix}$$

Basic Transformations

Translation along X–Axis

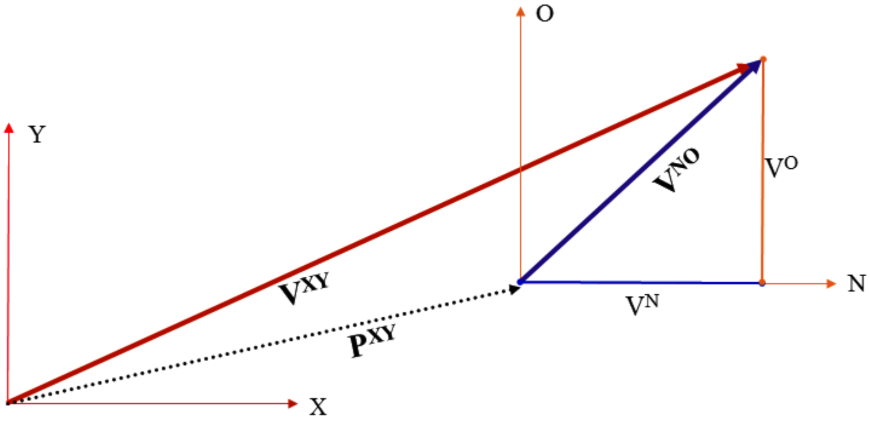


Writing v^{XY} in terms of v^{NO}

$$v^{XY} = \begin{bmatrix} p_x + v_n \\ v_o \end{bmatrix} = P + v^{NO}$$

Basic Transformations

Translation along XY–Axis



$$P = \begin{bmatrix} p_x \\ p_y \end{bmatrix}, \quad v^{XY} = P + v^{NO} = \begin{bmatrix} p_x + v_n \\ p_y + v_o \end{bmatrix}$$

Basic Transformations

2D Rotation

- Representing one coordinate frame in terms of another:

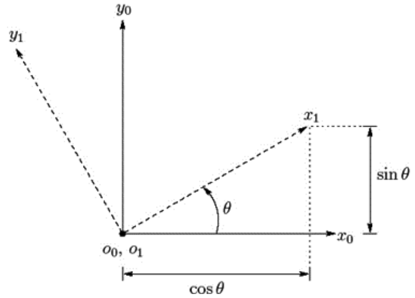
$$R_1^0 = \begin{bmatrix} x_1^0 & y_1^0 \end{bmatrix}$$

- unit vectors are defined as:

$$x_1^0 = |\vec{x_0}| = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}, \quad y_1^0 = |\vec{y_0}| = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

- This is called a **rotation matrix**



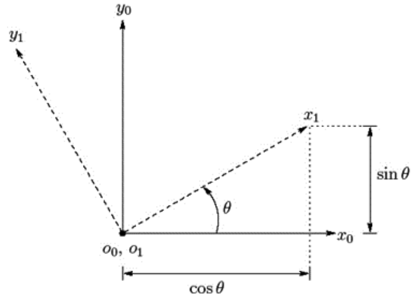
Rotations

Rotation matrices as projections

- Projecting the axes of O_1 onto the axes of frame O_0:

$$x_1^0 = \begin{bmatrix} (\vec{x_1} \cdot \vec{x_0}) \\ (\vec{x_1} \cdot \vec{y_0}) \end{bmatrix}, \quad y_1^0 = \begin{bmatrix} (\vec{y_1} \cdot \vec{x_0}) \\ (\vec{y_1} \cdot \vec{y_0}) \end{bmatrix}$$

$$\begin{aligned} R_1^0 &= \left[x_1^0(\theta) \,\middle|\, y_1^0(\theta) \right] = \begin{bmatrix} (\vec{x_1} \cdot \vec{x_0}) & (\vec{y_1} \cdot \vec{x_0}) \\ (\vec{x_1} \cdot \vec{y_0}) & (\vec{y_1} \cdot \vec{y_0}) \end{bmatrix} \\ &= \begin{bmatrix} |\vec{x_1}||\vec{x_0}| \cos\theta & |\vec{y_1}||\vec{y_0}| \cos\left(\theta + \frac{\pi}{2}\right) \\ |\vec{x_1}||\vec{x_0}| \cos\left(\theta - \frac{\pi}{2}\right) & |\vec{y_1}||\vec{y_0}| \cos\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \end{aligned}$$



[0] $\vec{a} \cdot \vec{b} = a_1\, b_1 + a_2\, b_2 + a_3\, b_3 = |\vec{a}|\, |\vec{b}|\, \cos\left(\widehat{\vec{a}, \vec{b}}\right)$

[0] $\sin(\theta) = -\sin(-\theta) = -\cos(\theta + 90^\circ) = \cos(\theta - 90^\circ), \quad \cos(\theta) = \cos(\pm\theta) = \sin(\theta + 90^\circ) = -\sin(\theta - 90^\circ)$

Rotations

Properties of rotation matrices

Rotation matrix has a number of interesting properties:

- Inverse Rotations:

$$\left(R_1^0\right)^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^{-1} = \frac{1}{det\left(R_1^0\right)} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \left(R_1^0\right)^T$$

- using odd/even properties (another interpretation)

$$R_0^1 = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \left(R_1^0\right)^T$$

[0] $\sin(\theta) = -\sin(-\theta) = -\cos(\theta + 90^\circ) = \cos(\theta - 90^\circ)$, $\cos(\theta) = \cos(\pm\theta) = \sin(\theta + 90^\circ) = -\sin(\theta - 90^\circ)$

Rotations

Properties of rotation matrices

- Columns (rows) of R are mutually **orthogonal**
- Each column (row) of R is a unit vector
- The **determinant** of a rotation matrix is always ± 1
  - $+1$ if we only use **right-hand convention**

$$det\,(R) = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos^2(\theta) + \sin^2(\theta) = 1$$

- The set of all $n \times n$ matrices (X) that satisfies these properties are called the **Special Orthogonal Group** of order n

$$X^{-1} = X^T \Rightarrow XX^T = \mathbf{I}_n, \quad det(XX^T) = 1 = det(X)det(X^T) = det(X)^2$$
$$X \in \mathcal{O}(n).\ \text{If } detX = 1 \Rightarrow X \in \mathcal{SO}(n)$$

[0] $\cos^2(\theta) + \sin^2(\theta) = 1$

Rotations

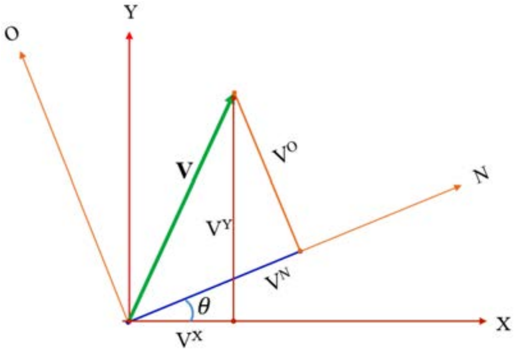
Rotation around the Z-Axis

$\theta =$ Angle of rotation between the XY and NO frames

$$v^{XY} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}, \quad v^{NO} = \begin{bmatrix} v_N \\ v_O \end{bmatrix}$$

$$v^{XY} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} v_N \\ v_O \end{bmatrix}$$

$$v^{XY} = R_{NO}^{XY}\, v^{NO}$$



Rotations

Rotation Around Major Axes

- Rotation around the Z-Axis

$$R_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Rotation around the Y-Axis

$$R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

- Rotation around the X-Axis

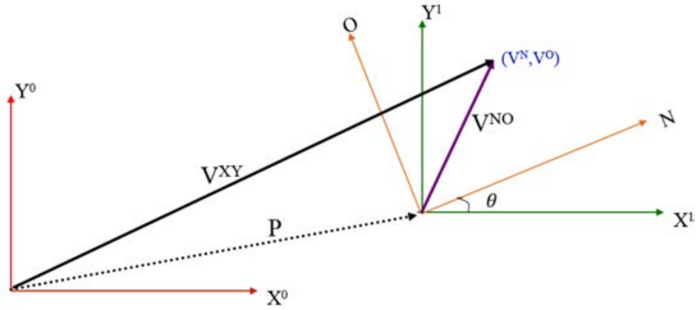
$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Frame Transformation

Translation Followed by Rotation

- **Translation** along P followed by **rotation** by θ around Z-axis

$$v^{XY} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} v_N \\ v_O \end{bmatrix}$$



- p_x, p_y are relative to the original coordinate frame.
- Translation followed by rotation is different than rotation followed by translation.
- knowing the coordinates of a point (V_N, V_O) in some coordinate frame $\{NO\}$ you can find the position of that point relative to another coordinate frame $\{X_0, Y_0\}$.

Thanks for your attention.

Questions?

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Robotics Research Interest Group (zuR^2IG)
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