Zagazig University, Faculty of Engineering
Academic year: 2015-2016
Specialization: Computer and Systems
Course Name: Selective Course (5)
Course Code : CSE4316
Examiners: Dr.\ Mohammed Nour

Mid-term Exam

Date: 9/1/2016
Exam Time: 45 Min.
No. of pages: 6
No. of Questions: 5
Full Mark: 80

a) Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the questions.	
b) The exam is in **6 pages**. Page 6 contains supplementary material that may be needed.	
c) Please show all work. **Intermediate steps must be legible to receive credit**.	
No.	**الاسم :**

Problem 1 [12 Marks]

For each of the following statements:

(a) Check (✓) for true or (✗) for the false.

(b) Give short comment for the correct one and correct the false one.

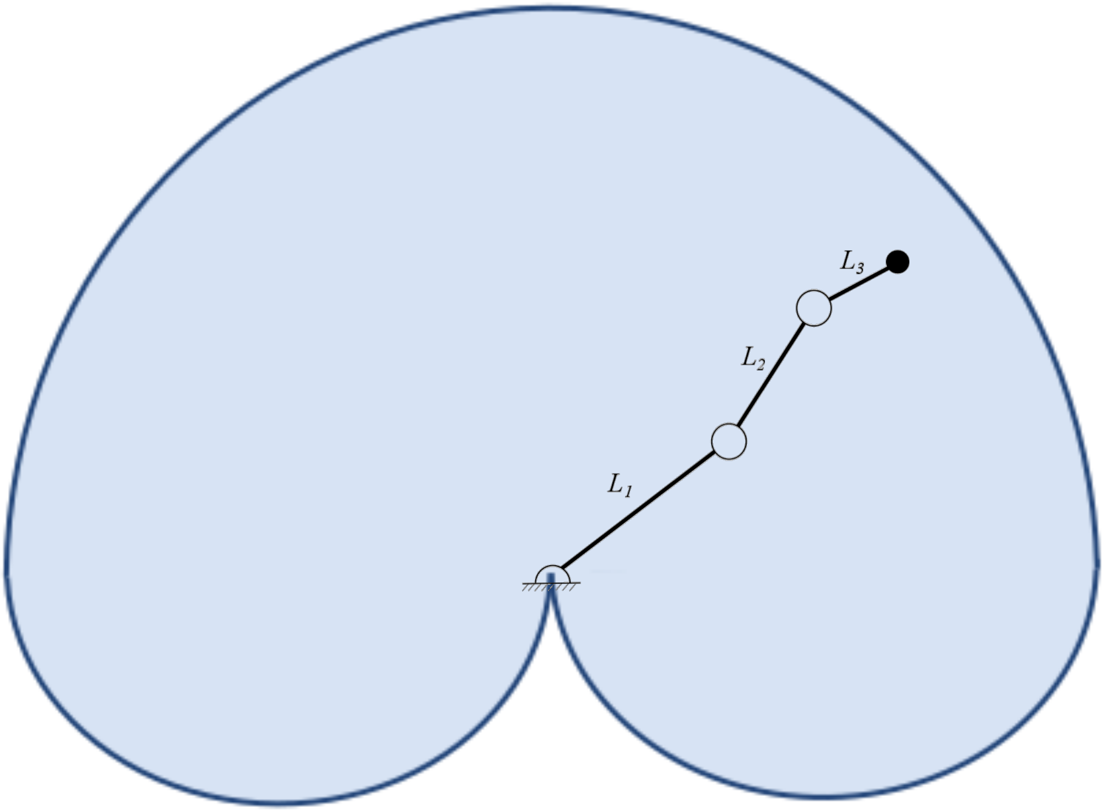
1. An example of the robot anthropomorphic characteristics is its kinematics.	✗
example of the robot anthropomorphic characteristics: **mechanical arm, sensors to respond to input, Intelligence to make decisions**.	
2. A rover is a kind of industrial robots.	✗
A rover is a kind of **mobile** robots.	
3. The inverse of the rotation matrix is its transpose.	✓
The inverse of the rotation matrix is its transpose **as one of its properties:** $R^{-1} = R^T$	
4. The dimension of SO(3) is 3.	✓
The dimension of SO(3) is 3 as **Special Orthogonal group of order *n* includes matrices of dimension *n x n*** $\mathrm{R}_{n\times n} \in \mathrm{SO(n)}$	
5. If r_1 and r_2 are two rows in a rotation matrix, then $r_1 r_2^T = 0$.	✓
as **rotation matrix columns (rows) are mutually orthogonal (i.e. its dot product is zero).** For example: $R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ $r_1 r_2^T = -\cos(\theta)\sin(\theta) + \cos(\theta)\sin(\theta) = 0$	

Problem 2 [2+10= 12 Marks]

For the three link manipulator shown in figure:

a) What is the term for the set of all points that the end effector can reach?
Robot workspace or Robot Work envelope .

b) Draw the set of all points that the end effector can reach where the base joint angle is limited to ±180°, *L1 > L2 > L3* and *L2 + L3 > L1*.



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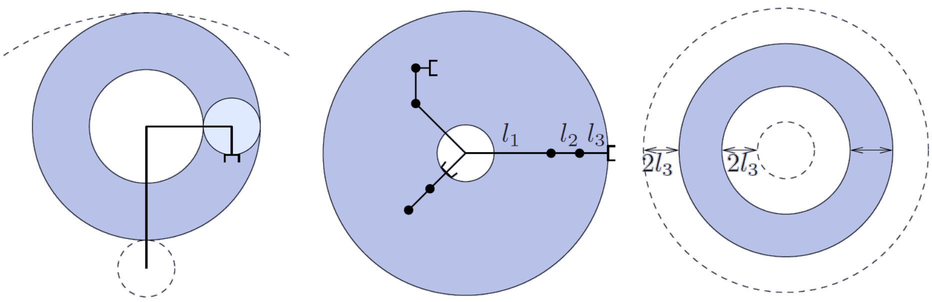
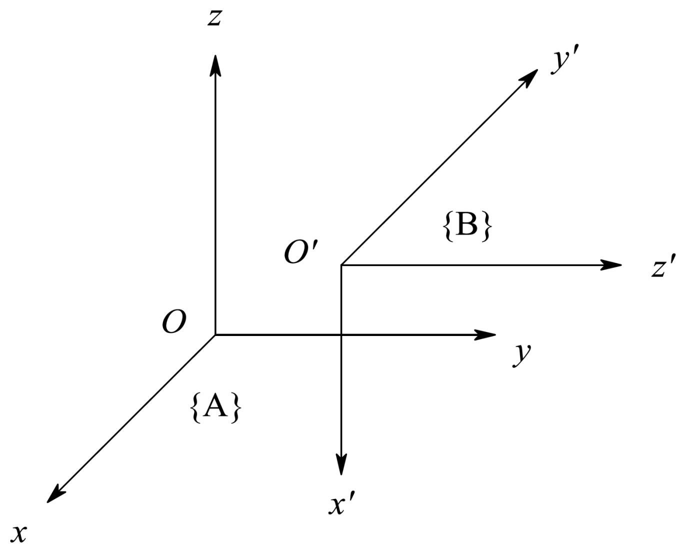


construction reachable workspace dextrous workspace

Note: due to the typo in *L2 + L3 > L1 (it was meant to be L2 + L3 < L1), if the answer is given as a full disk, it is also accepted as a correct answer.*

Problem 3 [4+4+4+4= 16 Marks]

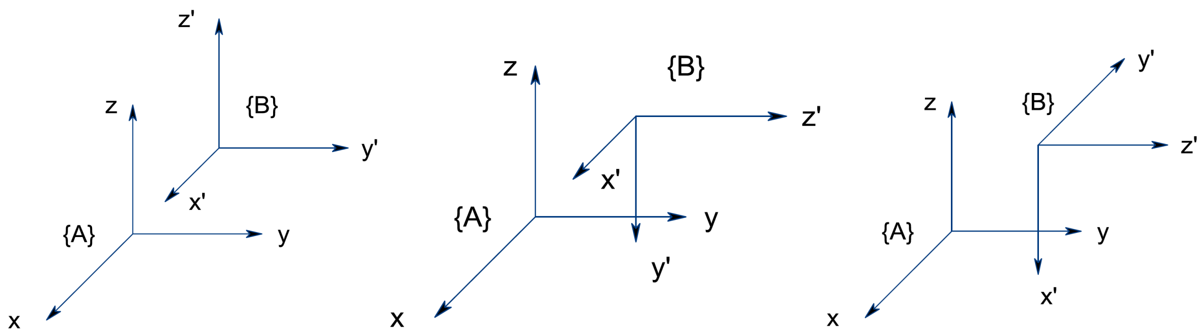
Two coordinate frames *A(x, y, z)* and *B(x', y', z')* are shown below. The origin of *{B}* with respect to *{A}* is given by $[1\ 2\ 3]^T$



Find H_B^A (i.e. ^{A}B, the homogeneous transformation matrix to represent B w.r.t. A).

Alt.1:

Assume that initially the frames are coincident. First **translate** B 1 unit in x, 2 units in y and 3 units in z axis. Then, **Rotate** about x' -90°. Then **Rotate** about z' 90°.



Therefore:

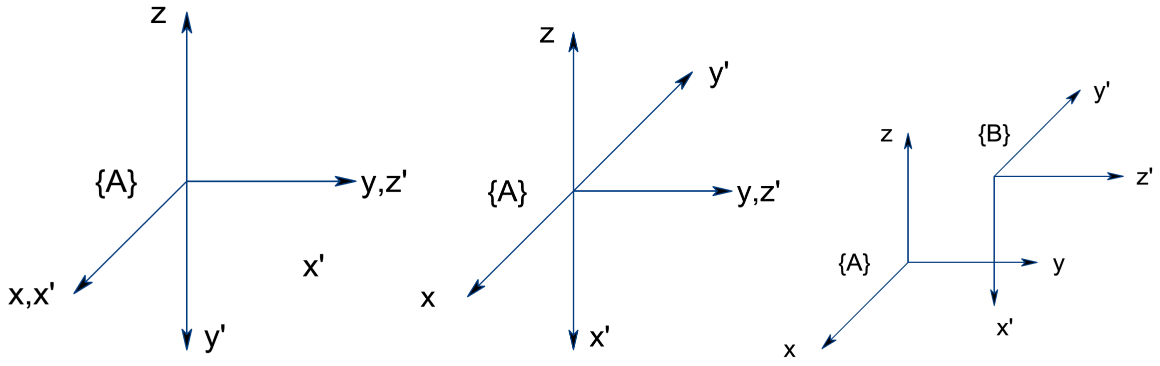
$$^A\mathrm{B} = \mathrm{Trans}(1{,}2{,}3)\ \mathrm{R_X}(-90)\ \mathrm{R_Z}(90)$$

$$^AB = \begin{bmatrix}1\\2\\3\end{bmatrix} \oplus \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & -1 & 0\end{bmatrix} * \begin{bmatrix}0 & -1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix} \oplus \begin{bmatrix}0 & -1 & 0\\0 & 0 & 1\\-1 & 0 & 0\end{bmatrix}$$

$$H_B^A = \begin{bmatrix}0 & -1 & 0 & 1\\0 & 0 & 1 & 2\\-1 & 0 & 0 & 3\\0 & 0 & 0 & 1\end{bmatrix}$$

Alt.2:

First Rotate about x -90°. Then rotate about y 90°. Then translate 1 unit in x, 2 units in y and 3 units in z axis.



Therefore:

$$^{A}\mathrm{B} = \mathrm{R_Y}(90)\,\mathrm{R_X}(-90)\,\mathrm{Trans}(1,2,3)$$

$$^{A}B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$H_B^A = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

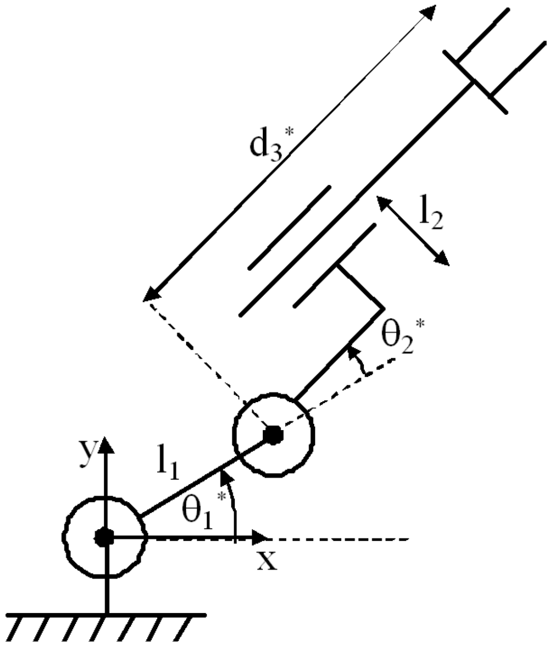
Problem 4 [2+18 = 20 Marks]

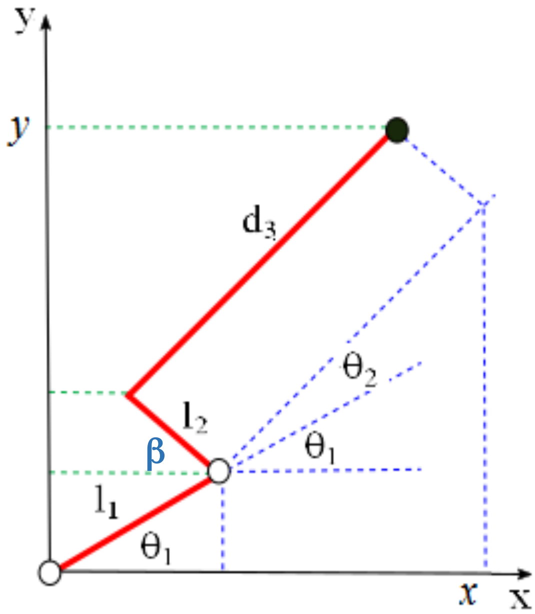
A planar 2–link RRP robot is given in the figure below.

a) What are its configuration space variables?

configuration space variables are : $[\theta_1 \quad \theta_2 \quad d_3]^T$…………………

b) Find its forward kinematics





Note that

$\beta = 180° - [90° + \theta_1 + \theta_2] = 90° - (\theta_1 + \theta_2)$

$$\sin(\beta) = \sin[90° - (\theta_1 + \theta_2)] = \cos[-(\theta_1 + \theta_2)] = \cos(\theta_1 + \theta_2) = C_{12}$$
$$\cos(\beta) = \cos[90° - (\theta_1 + \theta_2)] = \sin[-(\theta_1 + \theta_2)] = -\sin(\theta_1 + \theta_2) = -S_{12}$$

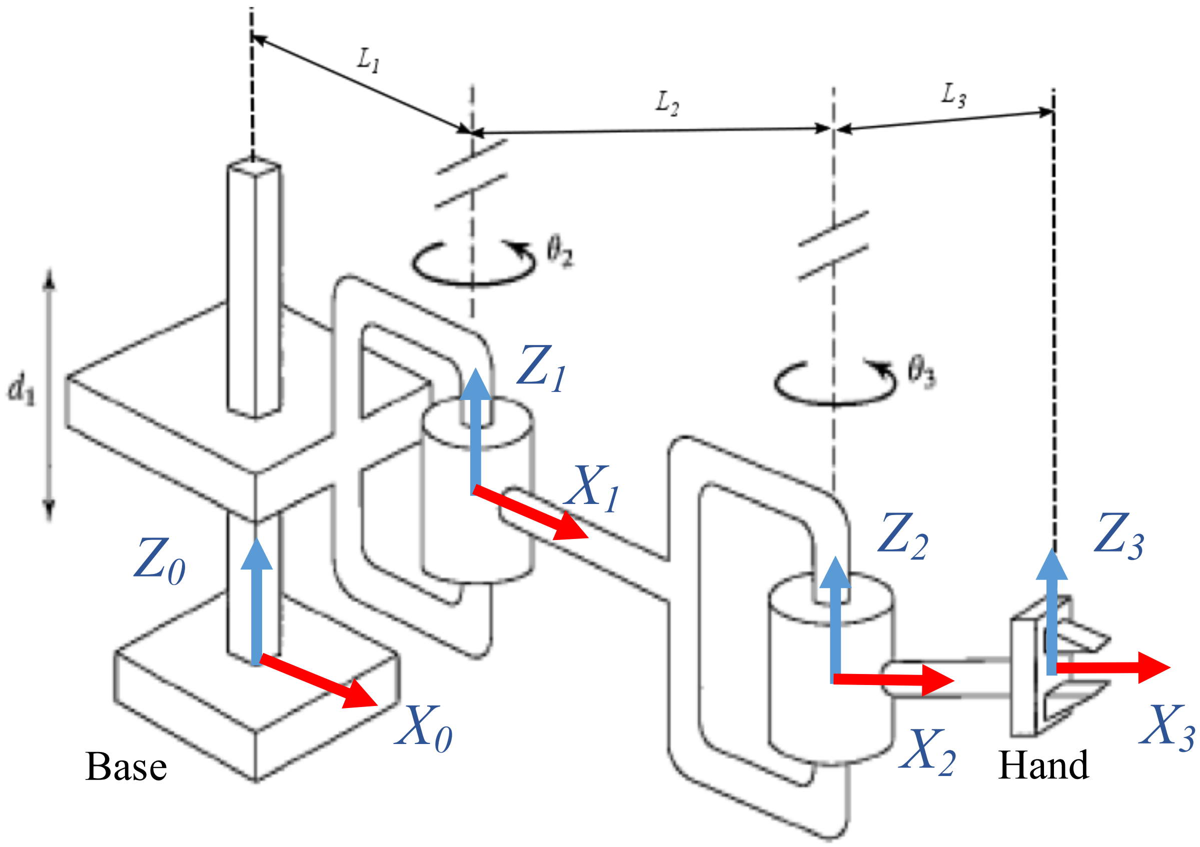
$$x = L_1 \cos\theta_1 - L_2 \sin(\theta_1 + \theta_2) + d_3 \cos(\theta_1 + \theta_2)$$
$$y = L_1 \sin\theta_1 + L_2 \cos(\theta_1 + \theta_2) + d_3 \sin(\theta_1 + \theta_2)$$

$$H_3^0 = \begin{bmatrix} c_1 - s_1 & 0 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = H_3^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$
$$X = L_1 C_1 - L_2 S_{12} + d_3 C_{12}$$
$$Y = L_1 S_1 - L_2 C_{12} + d_3 S_{12}$$

Problem 5 [4+6+3+7= 20 Marks]

For the three-link PRR manipulator shown in the following figure
(a) Assign appropriate frames for D-H representation (draw them on the figure).
(b) Fill out the D-H parameters table.
(c) Write all the *A* matrices.
(d) Write the H_H^0 (hand frame relative to base frame) in terms of the *A* matrices.

(a)



(b)

Link	*θ*	*d*	*a*	*α*
1	*0*	d_1	L_1	*0*
2	θ_2	*0*	L_2	*0*
3	θ_3	*0*	L_3	*0*

(c) Using the given:

$$A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and substituting with parameter values as given in the DH-parameter table, we get:

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & \ell_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & \ell_2 C_2 \\ S_2 & C_2 & 0 & \ell_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & \ell_3 C_3 \\ S_3 & C_3 & 0 & \ell_3 S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d) $$H_H^0 = A_1 A_2 A_3 = \begin{bmatrix} C_{23} & -S_{23} & 0 & \ell_2 C_2 + \ell_3 C_{23} + \ell_1 \\ S_{23} & C_{23} & 0 & \ell_2 S_2 + \ell_3 S_{23} \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Supplementary Material

Note: you may need **some or none** of these identities:

$$R_X(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix},$$

$$R_Y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix},$$

$$R_Z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sin\theta = -\sin(-\theta) = -\cos(\theta + 90^\circ) = \cos(\theta - 90^\circ),$$
$$\cos\theta = \cos(-\theta) = \sin(\theta + 90^\circ) = -\sin(\theta - 90^\circ).$$

$$\cos(\theta_1 \pm \theta_2) = c_{12} = c_1 c_2 \mp s_1 s_2,$$
$$\sin(\theta_1 \pm \theta_2) = s_{12} = c_1 s_2 \pm s_1 c_2,$$

$$c^2\theta + s^2\theta = 1.$$

$$A^2 = B^2 + C^2 - 2BC\cos a.$$

$$\sin(\alpha \mp \beta) = \sin\alpha\cos\beta \mp \sin\beta\cos\alpha$$
$$\cos(\alpha \mp \beta) = \cos\alpha\cos\beta \pm \sin\beta\sin\alpha$$
$$if \quad \cos\theta = b \quad then \quad \theta = A\tan 2(\pm\sqrt{1-b^2}, b)$$
$$if \quad \sin\theta = b \quad then \quad \theta = A\tan 2(b, \pm\sqrt{1-b^2})$$

$$if \quad a\cos\theta + b\sin\theta = c \quad then \quad \theta = A\tan 2(b,a) + A\tan 2(\pm\sqrt{a^2+b^2-c^2}, c)$$
$$if \quad a\cos\theta - b\sin\theta = 0 \quad then \quad \theta = A\tan 2(a,b) \quad and \quad \theta = A\tan 2(-a,-b)$$