**Zagazig University,Faculty of Engineering** **Midterm Exam** **Date**: 27/04/2017
**Academic Year**: 2016/2017 **Exam Time**: 75 Minutes
**Specialization**: Computer & Systems Eng. **No. of Pages**: 7
**Course Name**: Elective Course (5) **No. of Questions**: 3
**Course Name**: CSE4316:Robotics **Full Mark**: [ 40 ]
**Examiner**: Dr. **Mohammed Nour**

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▷ Please **answer all questions**. Use **3** decimal digits approximation.

▷ Mark your **answers** for all questions **in the Answer Sheet** provided.

▷ In last page, some supplementary identities you *may* need.

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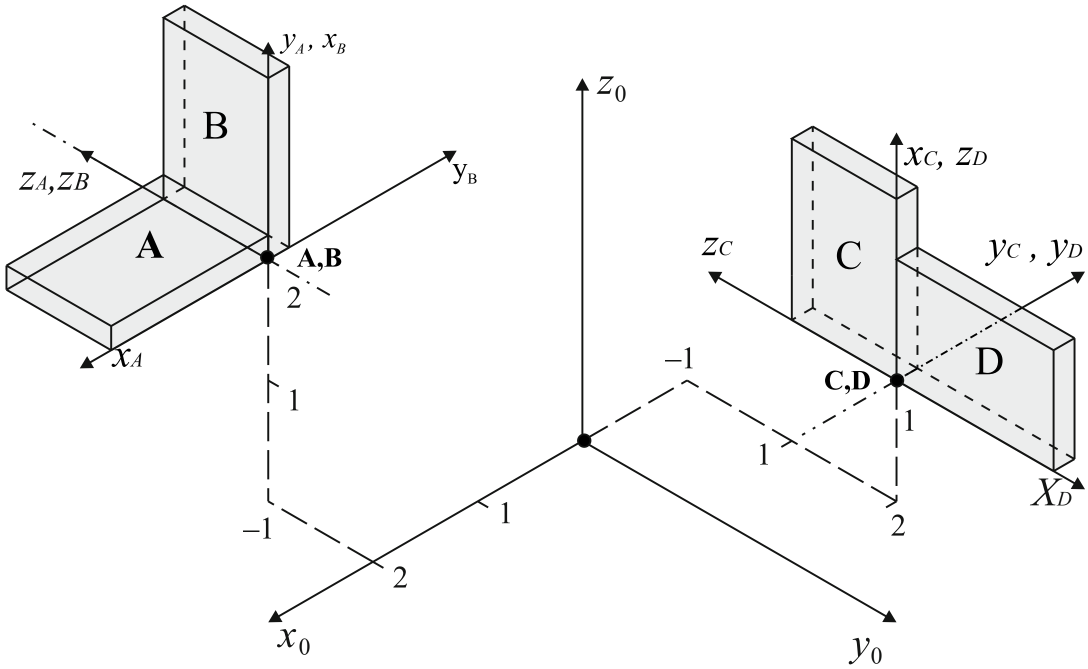
**Question 1.** [ 10 Marks ] ($2 \times 5$)

1. Which of the following is **false** for a **parallel** manipulator
   a) it forms a closed-chain.
   b) Gruebler's formula may be applied to count its DOF
   c) its configuration space is always planar.
   d) it is two or more series chains connect the end–effector to the base.
2. Degrees of Freedom of manipulator are Number of
   a) position variables that have to be specified. b) configuration parameters minimally specified.
   c) of its joints. d) dimensions of its workspace.
3. One of the robot anthropomorphic characteristics is its
   a) mechanical arm b) degrees of freedom c) mobility as in rovers d) substitution for humans
4. A suitable robot configuration for high precision pick and place operations is $\cdots$
   a) SCARA b) Articulated-arm c) cylindrical d) Cartesian
5. The world reference frame is
   a) a universal fixed coordinate frame
   b) used to specify movements of each individual joint of the robot.
   c) specifies the movements of the robot tool–tip w.r.t hand frame.
   d) specifies the movements of work object w.r.t station frame.

**Question 2.** [ 15 Marks ] ($3 \times 5$)

Consider the pose of the objects $A, B, C$ and $D$ in space, as shown next:




6. The **relative** rotation matrix that displaces object A into the new pose **B** is:

a) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. The **relative** transformation that displaces object B into the new pose **C** is:

a) $Trans(-1, 3, -3)$ b) $Rot(y_B, 90°)\,Trans(-1, 2, 1)$

c) $Trans(-1, 2, 1)$ d) $Rot(y_0, -90°)\,Trans(2, -1, 1)$

8. The homogeneous transformation that relates frame $A$ to frame 0 (i.e. $H^0_A$) is:

a) $\begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

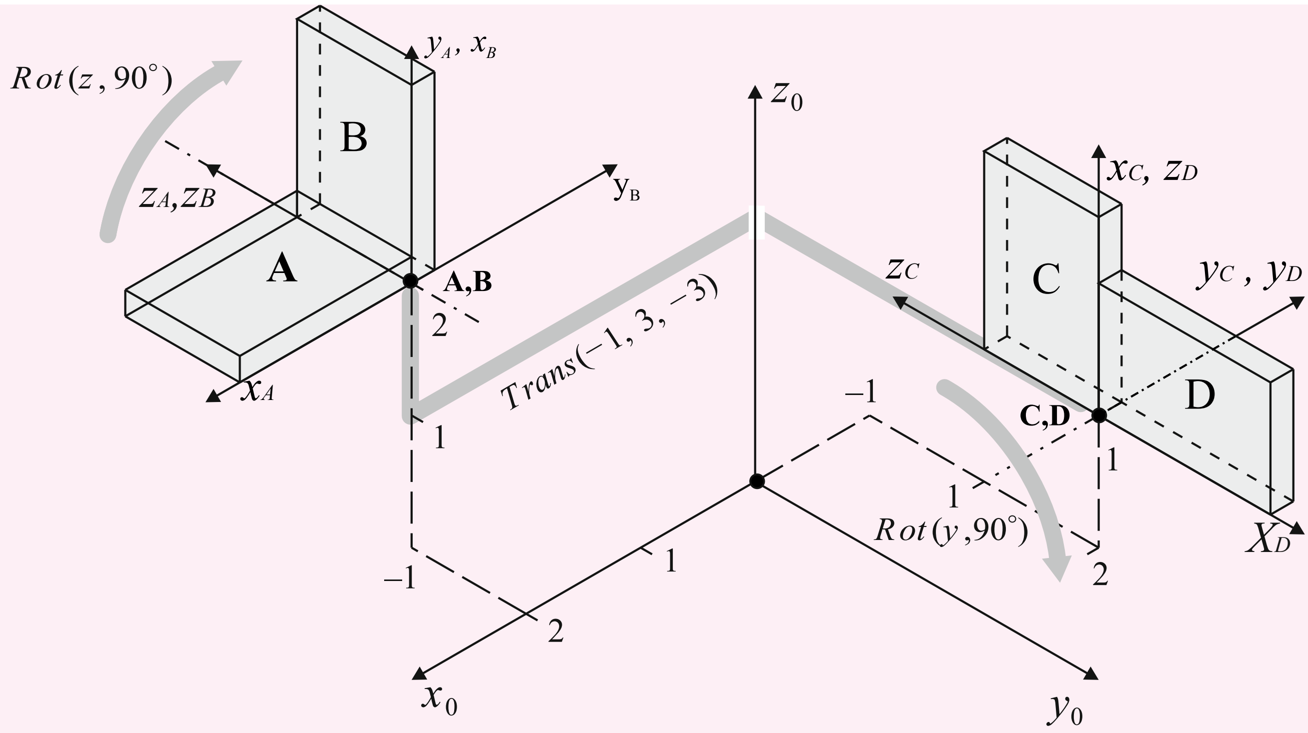
9. The homogeneous transformation that relates frame $D$ to frame $A$ (i.e. $H^A_D$) is:

a) $\begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 2 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -1 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

10. The frame transformation $H^0_D$ can be expressed as:

a) $\begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

**Solution**




to rotate the coordinate frame $\{A\}$ for $90^\circ$ in the counter-clockwise direction around the $z$–axis. This can be achieved by:

$$R_B^A = Rot(z_A, 90^\circ) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The displacement resulted in a new pose of the object and new frame $\{x_B, y_B, z_B\}$ shown in Figure. We shall displace this new frame for $-1$ along the $x_B$ axis, 3 units along $y_B$ axis and $-3$ along $z_B$ axis, so:

$$T_2 = Trans(-1, 3, -3)$$

This frame will be finally rotated for $90^\circ$ around the $y_C$ axis in the positive direction

$$R_3 = Rot(y_C, 90^\circ) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

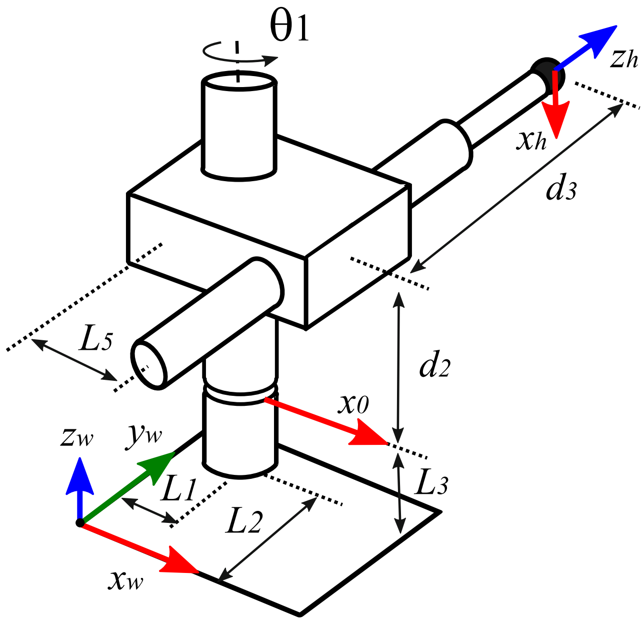
$$H_A^0 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_D^A = R_1\, T_2\, R_3 = Rot(z_A, 90^\circ)\, Trans(-1, 3, -3)\, Rot(y_C, 90^\circ)$$
$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

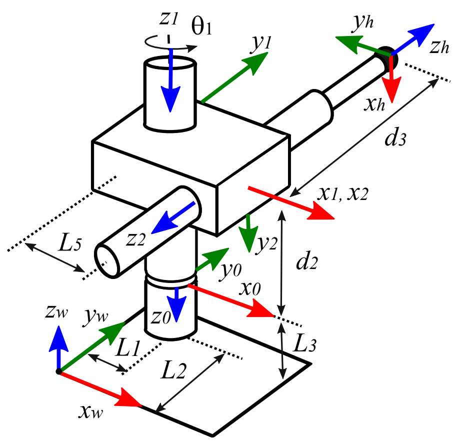
$$H_D^0 = H_A^0\,H_D^A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Question 3.** [ 15 Marks ] $(2+3+2+4+4)$

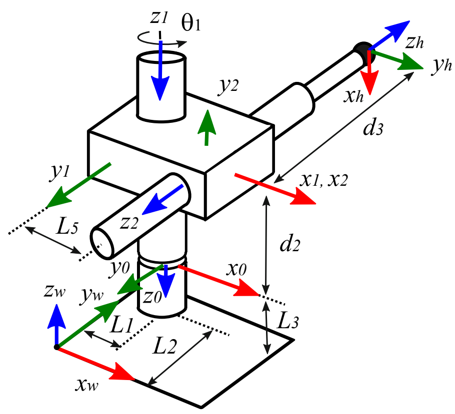
Consider the **R2P** robot manipulator mechanism shown next with $\{w\}$, $\{0\}$ and $\{h\}$ as the world, base and tool frame, respectively:



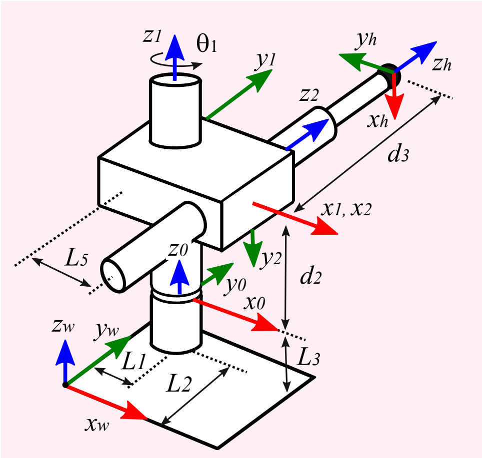

11. According to the DH conventions, we can assign frames to the joints as:

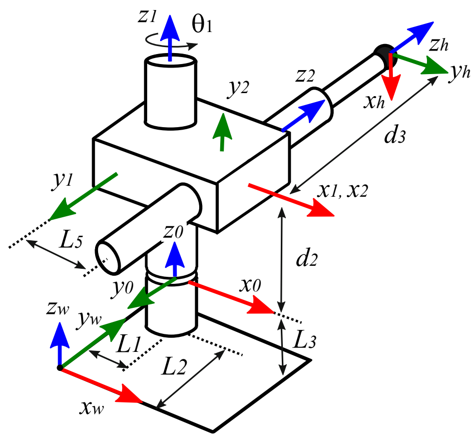



a)




b)

c) 


d) 


12. The DH parameters of the **spherical** wrist joints:

a)

| Link | $a_i$ | $\alpha_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | $L_2$ | 0 | 0 | $\theta_1^*$ |
| 2 | $L_5$ | 0 | $d_2^*$ | 0 |
| 3 | 0 | 0 | $d_3^*$ | 0 |

b)

| Link | $a_i$ | $\alpha_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $\theta_1^*$ |
| 2 | $L_5$ | 90 | $d_2^*$ | 0 |
| 3 | 0 | 0 | $d_3^*$ | −90 |

c)

| Link | $a_i$ | $\alpha_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $\theta_1^*$ |
| 2 | 0 | 0 | $d_2^*$ | 90 |
| 3 | $L_3$ | 0 | $d_3^*$ | 0 |

d)

| Link | $a_i$ | $\alpha_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $\theta_1^*$ |
| 2 | $L_5$ | −90 | $d_2^*$ | 0 |
| 3 | 0 | 0 | $d_3^*$ | 90 |

13. the homogeneous transformation matrix $A_2$ is found as:

a) $\begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
b) $\begin{bmatrix} 1 & 0 & 0 & L_5 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
c) $\begin{bmatrix} 0 & -1 & 0 & L_2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & L_3 \\ 0 & 1 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

14. The robot tool–tip coordinates can be expressed in the base frame using $T_h^0$ that is calculated as:

a) $\begin{bmatrix} 0 & -c_1 & -s_1 & -s_1 d_3 + c_1 L_3 \\ 0 & s_1 & c_1 & c_1 d_3 + s_1 L_3 \\ -1 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
b) $\begin{bmatrix} 0 & c_1 & s_1 & -s_1 d_3 + c_1 \\ 0 & -s_1 & -c_1 & c_1 d_3 + s_1 \\ 1 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

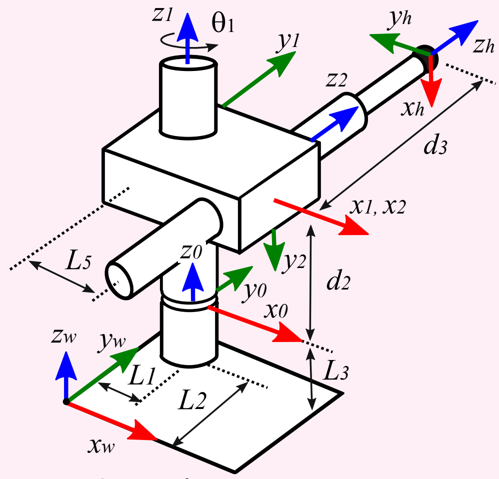
c) $\begin{bmatrix} 0 & s_1 & c_1 & c_1 d_3 + s_1 + L_1 \\ 0 & -c_1 & -s_1 & s_1 d_3 + c_1 + L_2 \\ -1 & 0 & 0 & d_2 + L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
d) $\begin{bmatrix} 0 & -c_1 & -s_1 & -s_1 d_3 + c_1 L_5 \\ 0 & s_1 & c_1 & c_1 d_3 + s_1 L_5 \\ -1 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

15. At the configuration $\mathbf{q} = [0°, L_4, L_6]^T$, the hand **pose** at this configuration is calculated as:

a) $\begin{bmatrix} 0 & 1 & 0 & L_1 \\ 0 & 0 & -1 & L_6 \\ -1 & 0 & 0 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
b) $\begin{bmatrix} 0 & 1 & 0 & L_3 + L_4 \\ 0 & 0 & 1 & L_2 + L_5 \\ 1 & 0 & 0 & L_1 + L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
c) $\begin{bmatrix} 0 & -1 & 0 & L_3 \\ 0 & 0 & 1 & L_2 \\ -1 & 0 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
d) $\begin{bmatrix} 0 & -1 & 0 & L_5 + L_1 \\ 0 & 0 & 1 & L_6 + L_2 \\ -1 & 0 & 0 & L_4 + L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

**Solution**

According to the DH conventions, we can assign frames to all joints as:




The DH parameters of the robot joints:

| Link | $a_i$ | $\alpha_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $\theta_1^*$ |
| 2 | $L_5$ | $-90$ | $d_2^*$ | 0 |
| 3 | 0 | 0 | $d_3^*$ | 90 |

The matrices describing the relative poses of the neighboring coordinate frames:

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_2 = \begin{bmatrix} 1 & 0 & 0 & L_5 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_3 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The geometric model of the robot arm is represented by the product of first three matrices:

$$H_3^0 = A_1\, A_2\, A_3 = \begin{bmatrix} 0 & -c_1 & -s_1 & -s_1 d_3 + c_1 L_5 \\ 0 & s_1 & c_1 & c_1 d_3 + s_1 L_5 \\ -1 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that $T_h^0 = H_3^0$ i.e. this transformation expresses the robot tool tip coordinates relative to the robot base frame.

The homogeneous transformation from the robot tool tip to the world frame:
Note that the transformation from the robot base frame to the world frame is a pure translation $Trans(L_1, L_2, L_3)$.
Therefore we can get $T_h^w$ by **simple addition** of the translation vector $[L_1\ L_2\ L_3\ 1]^T$ to the last column of $H_3^0$:

$$T_h^w = \begin{bmatrix} 0 & -c_1 & -s_1 & -s_1 d_3 + c_1 L_5 + L_1 \\ 0 & s_1 & c_1 & c_1 d_3 + s_1 L_5 + L_2 \\ -1 & 0 & 0 & d_2 + L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Another way to **systematically** calculated the homogeneous transformation from the robot tool tip to the world frame as:

$$T_h^w = H_0^w\, H_3^0 = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & L_2 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -c_1 & -s_1 & -s_1 d_3 + c_1 L_5 \\ 0 & s_1 & c_1 & c_1 d_3 + s_1 L_5 \\ -1 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -c_1 & -s_1 & -s_1 d_3 + c_1 L_5 + L_1 \\ 0 & s_1 & c_1 & c_1 d_3 + s_1 L_5 + L_2 \\ -1 & 0 & 0 & d_2 + L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The given configuration $\mathbf{q} = [0, L_4, L_6]^T$ corresponds to $\mathbf{q} = [\theta_1, d_2, d_3]^T$. So, the hand pose at this configuration is calculated by direct substitution of $q$ in $T_h^w$:

$$T_h^w = \begin{bmatrix} 0 & -1 & 0 & L_5 + L_1 \\ 0 & 0 & 1 & L_6 + L_2 \\ -1 & 0 & 0 & L_4 + L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Supplementary Material

Note: you *may* need some or none of these identities:

$$R_Z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_Y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}, \quad R_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$

$$\sin(\theta) = -\sin(-\theta) = -\cos(\theta + 90^\circ) = \cos(\theta - 90^\circ)$$

$$\cos(\theta) = \cos(-\theta) = \sin(\theta + 90^\circ) = -\sin(\theta - 90^\circ)$$

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

**if** $\cos(\theta) = b$, **then** $\theta = \text{atan2}\left(\pm\sqrt{1 - b^2}, b\right)$

**if** $\sin(\theta) = b$, **then** $\theta = \text{atan2}\left(b, \pm\sqrt{1 - b^2}\right)$

**For a triangle:** $A^2 = B^2 + C^2 - 2BC\cos(a)$, $\frac{\sin(a)}{A} = \frac{\sin(b)}{B} = \frac{\sin(c)}{C}$

**if** $a\cos(\theta) + b\sin(\theta) = c$, **then** $\theta = \text{atan2}(b, a) + \text{atan2}\left(\pm\sqrt{a^2 + b^2 - c^2}, c\right)$

**if** $a\cos(\theta) - b\sin(\theta) = 0$, **then** $\theta = \text{atan2}(a, b) + \text{atan2}(-a, -b)$