CSE421: Digital Control

Sheet 1

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Exercise 1.

Draw a block diagram of digital feedback control system. Show the type of the signal at each point on the block diagram whether continuous, discrete and ZOH (output of DAC) signals.

Exercise 2.

What are the advantages of digital control over analog control systems?

Exercise 3.

Explain how to adjust the sampling interval using timer interrupts and ballast coding.

Exercise 4.

Solve the following difference equations:

a) $x(k+2) + 3x(k+1) + 2x(k) = 0; \quad x(0) = 0; \quad x(1) = 1$

b) $x(k+2) - x(k+1) - x(k) = 0; \quad x(0) = 1; \quad x(1) = 1$

c) $x(k+2) - 1.3x(k+1) + 0.4x(k) = 0; \quad x(0) = 1; \quad x(1) = 0$

Exercise 5.

Given the difference equation:

$$y_k = 0.5\,y_{k-1} + 0.5\,y_{k-2} + 0.25\,u_{k-1}$$

a) Find the discrete transfer function: $H(z) = \frac{Y(z)}{U(z)}$

b) Draw a block diagram of this discrete system using delays and gains.

c) Find the pulse response of the system.

Exercise 6.

Consider a discrete damped sinusoid:

$$e_k = e(k) = r^k \sin(k\,\theta) = 0.5^k \sin\left(k\frac{\pi}{2}\right)$$

Plot the signal and then find :

a) The z-transform of e_k,

b) The pole/zero locations.

c) The oscillation speed (number of samples/cycle).

Linear Discrete Systems: Sampling and Aliasing

Exercise 7.

Give the two poles $s = -1, -2$ find the equivalent poles in the z-plan.

- Which of these two poles is faster in response?
- Which is the more dominant?

Exercise 8.

The standard form of the continuous-domain transfer function for an underdamped system is

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Which has pole locations at:

$$s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

Give corresponding z-plane complex poles at $z = r\ e^{\pm j\theta}$. Find the equivalent damping ratio ξ and natural frequency ω_n of these discrete system poles in terms of r, θ, and sampling period T.

Exercise 9.

Consider a signal of frequency 10 Hz, sampled at a rate of $F_s = 50$ Hz.

What are the frequencies that appear at the sampled signal? Does the aliasing problem occur? Why?

Exercise 10.

Consider a signal of frequency 10 Hz, sampled at a rate of $F_s = 15$ Hz.

What are the frequencies that appear at the sampled signal? Does the aliasing problem occur? Why? How to solve it?

Exercise 11.

Consider a signal of frequency 10 Hz, sampled at a rate of $F_s = 50$ Hz. Assume that there is a noise component at 40 Hz.

What are the frequencies that appear at the sampled signal? Does the aliasing problem occur? Why? How to solve it?

Exercise 12.

Q12. Using a MATLAB program, verify the ideal reconstruction method:

$$r(t) = \sum_{k=-\infty}^{\infty} r(kT)\,\mathrm{sinc}\,\frac{\pi(t-kT)}{T} = \sum_{k=-\infty}^{\infty} r(kT)\frac{\sin(\pi(t-kT)/T)}{(\pi(t-kT)/T)}$$

For your sampled data, use 6 samples ($k = 0 \to 5$) of a 1 Hz sine wave with $T = 0.2$. Compute a full period of the reconstructed continuous signal at a much finer time step, such as 0.01 second. Compare the reconstructed signal using only 6 samples with the original continuous signal. Does it really work?

Exercise 13.

What are the advantages of Zero-Order-Hold (ZoH) reconstruction over thee reconstruction using the ideal low pass filter (LPF).