CSE421: Digital Control

Assignment 5

Controller Design

Q1. A second-order continuous-time system is required to have a damping ratio of 0.7, and a settling time of about 1 s.

(a) Find the system poles in the s-plane.
(b) Find the corresponding poles in the z-plane, using the mapping $z = e^{sT}$, if the input and the output of the system are sampled every 0.1 sec.

Q2. The open-loop transfer function of a plant is given by:

$$G(s) = \frac{e^{-4s}}{2s+1}$$

Which is to be preceded by a ZOH circuit.

(a) Design a dead-beat digital controller for the system. Assume that = 1 s.
(b) Draw the block diagram of the system together with the controller.
(c) Plot the time response of the system.

Q3. Repeat Q2 using a Dahlin controller in order to achieve a closed-loop first-order response with a time constant 1 sec. Plot the response and compare with the results obtained from the dead-beat controller.

Q4. The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{10}{s(s+10)}.$$

Note that G(s) is to be preceded by a ZOH. Assume that T = 1 s and design a controller so that the system response to a unit step input is

$$y(kT) = 0\ ,\ 0.4,\ 1,\ 1,\ \ldots$$

Q5. The open-loop transfer function of a system together with a zero-order hold is given by

$$HG(z) = \frac{0.2(z+0.8)}{z^2 - 1.5z + 0.5}.$$

Design a digital controller so that the closed-loop system will have $\zeta = 0.6$ and $w_d = 3$ rad/s. The steady-state error to a step input should be zero. Also, the steady state error to a ramp input should be 0.5. Assume that T = 0.2 s.

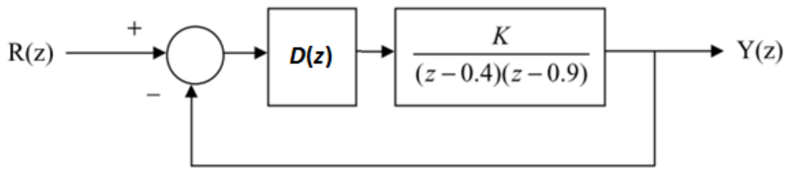
Q6. The open-loop transfer function of a system is

$$G(s) = \frac{e^{-0.16s}}{0.2s+1}$$

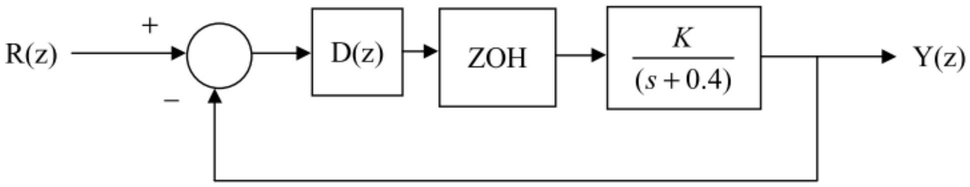
The system is preceded by a sampler and a zero-order hold. The closed-loop system is required to have a time constant of 0.4 s. assume the sample period is 0.04.

(a) Determine the digital controller D(z).
(b) Plot the unit step time response of the system with the controller.

Q7. Design a digital controller, D(z), such that the poles of the following closed-loop system are placed at z1,2 = 0.4 ± j0.4 in the z-plane. The steady-state error in the step response must be zero.

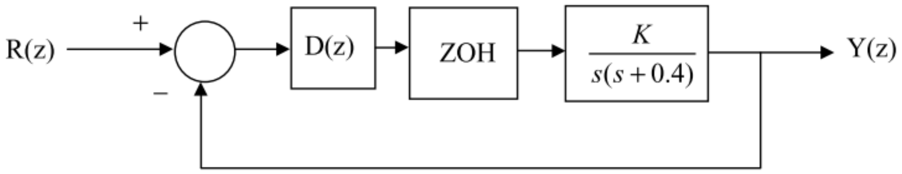


Q8. It is required to design a controller for the following system to achieve percent overshoot (PO) less than 20 %, settling time ts ≤ 10 s (2% criterion), and zero steady state error for step and ramp inputs. Assume that the sampling time is, T = 0.1 s and $K = 0.4$.



Derive the transfer function of the required digital controller.

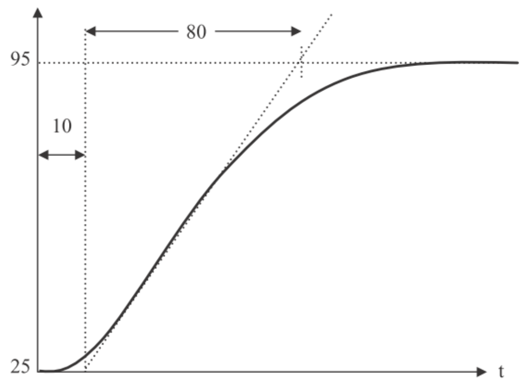
Q9. It is required to design a controller for the following closed-loop system such that percent overshoot (PO) less than 15 % and settling time ts ≤ 10 s (2% criterion), and zero steady state error for step and ramp inputs. Assume that the sampling time is, T = 0.2 s and $K = 0.4$.

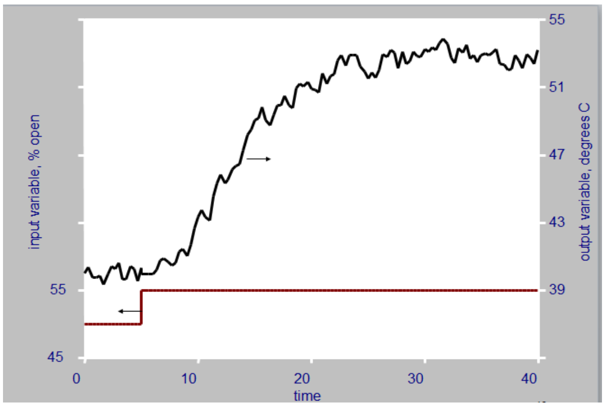


Derive the transfer function of the required digital controller.

Q10. The open-loop unit step responses of two systems are shown below. Obtain the transfer function of these systems and use the Ziegler–Nichols tuning algorithm to design discrete-time:

(a) proportional controller;
(b) PI controller;
(c) PID controller.





Q11. Explain what integral wind-up is when a PID controller is used. How can integral wind-up be avoided?

Q12. Explain what derivative kick is when a PID controller is used. How can derivative kick be avoided?

Q13. The continuous-time PI controller has the transfer function

$$\frac{U(s)}{E(s)} = \frac{K_p s + K_i}{s}.$$

Derive the equivalent discrete-time controller transfer function using the bilinear transformation:

$$s = \frac{2}{T}\frac{z-1}{z+1}.$$

Q14. A commonly used compensator in the s-plane is the lead lag, or lag lead with transfer function

$$\frac{U(s)}{E(s)} = \frac{s+a}{s+b}.$$

Find the equivalent discrete-time controller using the bilinear transformation.

Q15. Consider the following model

$$y(k) = ay(k-1) + bu(k-1) + e(k)$$

where y(k) is the system output at instant k, and e(k) is an equation error. Given the following input output data,

(a) deduce the matrix Φ.
(b) calculate the least squares estimate of a and b.
(c) calculate the model output.
(d) calculate the residuals.

t	**1**	**2**	**3**	**4**	**5**
***u*(*t*)**	**1**	**2**	**4**	**5**	**7**
***y*(*t*)**	**1**	**3**	**5**	**7**	**8**

Q16. Consider the following model

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + bu(k-1) + e(k)$$

where y(k) is the system output at instant k, and e(k) is an equation error. Given the same input output data in the previous question,

a) deduce the matrix Φ.
b) calculate the least squares estimate of a_1, a_2 and b.
c) calculate the model output.
d) calculate the residuals. Comment on the values of the residuals.