CSE421: Digital Control

Assignment 3

Discrete Block diagrams

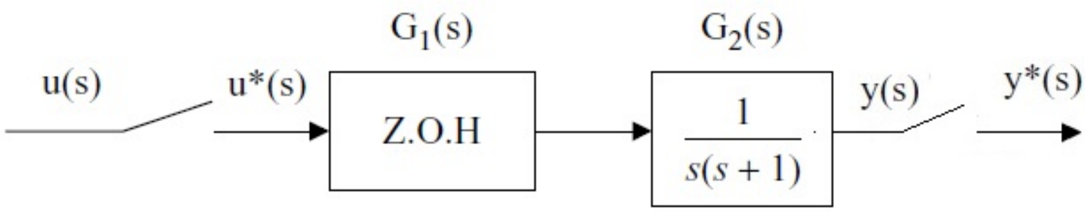
Q1. Assume that the following plant transfer functions are preceded by a zero-order hold. Compute the equivalent discrete transfer function G(z) using a sampling period T = 1. Check you answer with MATLAB.

$$(a)\ G(s) = \frac{1}{s^2}$$

$$(b)\ G(s) = \frac{1}{s(s+1)}$$

$$(c)\ G(s) = \frac{1}{s^2 - 1}$$

Q2. Calculate and plot the pulse response of the following system assuming that the sampling period T = 1sec.



Solution:

The transfer function of the ZOH is

$$G_1(s) = \frac{1 - e^{-Ts}}{s},$$

For this system, we can write

$$y(z) = G_1G_2(z)u(z).$$

Now, T=1 and

$$G_1G_2(s) = \frac{1-e^{-Ts}}{s^2(s+1)},$$

Or by partial fractional expansion we can write

$$G_1G_2(s) = (1-e^{-s})\left(\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}\right).$$

From the z-transform tables

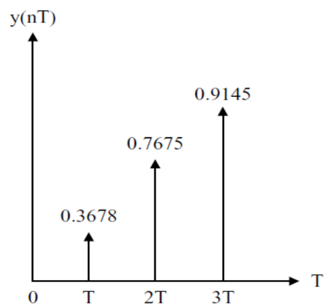
$$G_1G_2(z) = (1-z^{-1})\left(\frac{z}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-e^{-1}}\right) = \frac{ze^{-1}+1-2e^{-1}}{(z-1)(z-e^{-1})}.$$
$$= \frac{0.3678z + 0.2644}{z^2 - 1.3678z + 0.3678}.$$

For a pulse input, u(z) = 1. Therefore the pulse response will be given by

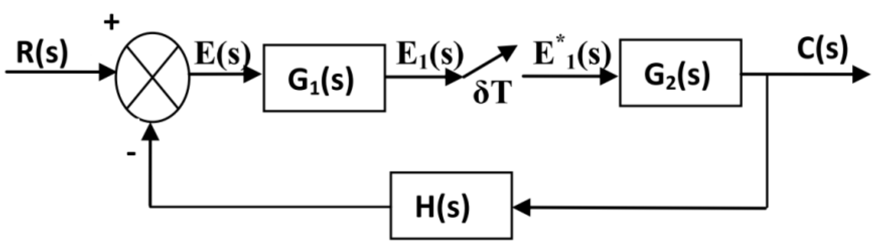
$$y(z) = G_1G_2(z)u(z) = G_1G_2(z) = \frac{0.3678z + 0.2644}{z^2 - 1.3678z + 0.3678}.$$

After long division, we obtain the time response

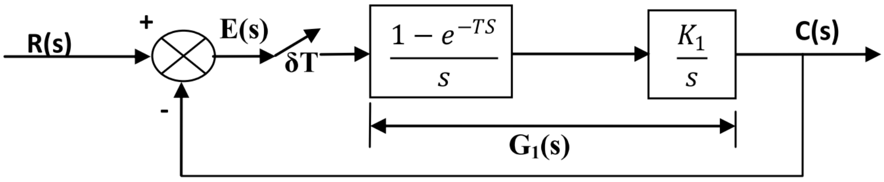
$$y(n) = 0.3678\delta(n-1) + 0.7675\delta(n-2) + 0.9145\delta(n-3) + \cdots$$



Q3. Obtain the output C(z) for the discrete-time control system whose block diagram is shown below:



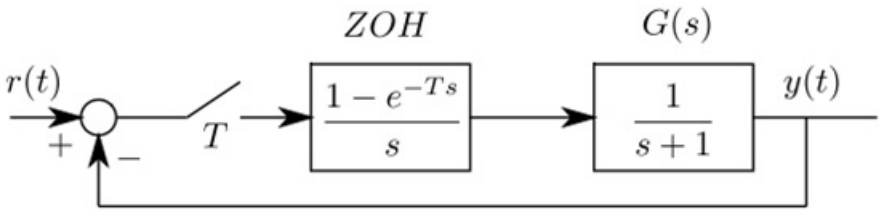
Q4. Obtain the output sequence c(kT) if the input r(t) is a unit step and T= 1sec for the discrete-time control system whose block diagram is shown below:



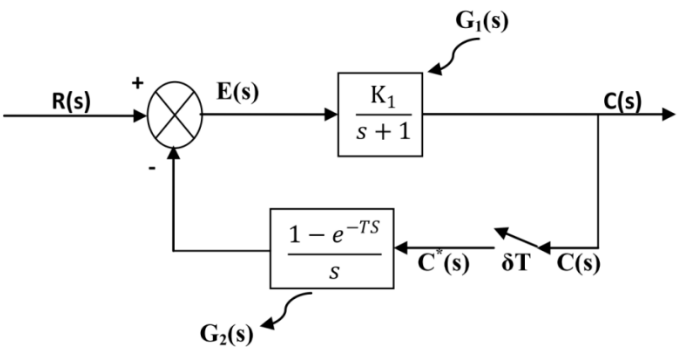
Final answer:

$$c(nT) = 1-(1-k_1)^n.$$

Q5. Sketch the step response y(t) of the system shown below for the three samples k = 0, 1, 2. Use sample period T = 1.



Q6. Obtain the output sequence c(kT) if the input r(t) is a unit step, T= 0.2 sec and K_1 = 1 for the discrete-time control system whose block diagram is shown below:



Final answer:

$$c(nT) = \frac{1}{2}[1-(0.6374)^n] = \{0,0.181,0.297,0.371,0.418,...\}$$