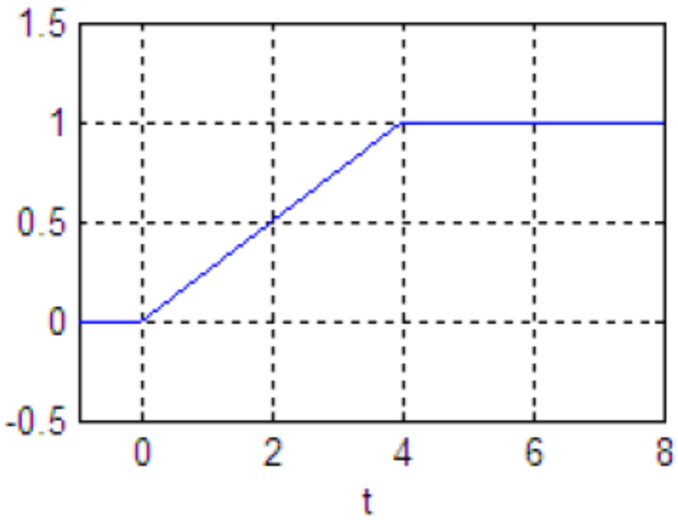
CSE421: Digital Control

Assignment 2

The z-Transform

Q1. Obtain the z-transform for the following curve (assume T=1):



Q2. A function y(t) = 2sin(4t) is sampled every T = 0.1 s. Find the z-transform of the resultant number sequence.

Q3. Find the z-transform of the following function, assuming that T = 0.5 sec:

$$(a)\quad Y(s)=\frac{1}{s^2(s+1)},\qquad (b)\quad Y(s)=\frac{1}{s^2},$$

$$(c)\quad Y(s)=\frac{e^{-Ts}}{s(s+1)},\qquad (d)\quad Y(s)=\frac{(s+3)}{(s+1)(s+2)},$$

$$(e)\quad Y(s)=\frac{(s+1)}{s(s+2)},\qquad (f)\quad Y(s)=\frac{s}{(s+1)^2}$$

Check your answer using the tables of z-transform.

Q4. Determine the z-transform of the following time domain functions.

$$(a) \quad x(k) = k$$

$$(b) \quad x(k) = k^2$$

$$(c) \quad x(t) = 1 - e^{-at}$$

$$(d) \quad x(t) = te^{-at}$$

Hint: you can check your answer with MATLAB command **ztrans**. For example, we can solve (d) using the following commands:

```
>> syms T a k z;
>> xk=(k*T)*(exp(-a*k*T));
>> xz=ztrans(xk,k,z)
```

Q5. For the discrete transfer function G(z) below:

$$G(z) = \frac{1}{z^2 - 0.5z + 0.5}$$

Find:

(a) The unit pulse response.
(b) The unit step response. Verify the DC gain.

Q6. Determine the final value of the sequences whose z-transform is:

$$X(z) = \frac{1}{(1 - z^{-1})} - \frac{1}{(1 - e^{aT} z^{-1})}$$

Q7. Find the inverse z-transform of $X(z)$ using both long division and partial fraction methods. Find also the steady state value of $x(n)$.

$$(a) \quad X(z) = \frac{10z + 5}{(z-1)(z-0.2)}, \qquad (b) \quad X(z) = \frac{1}{1+z}$$

$$(c) \quad X(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}, \qquad (d) \quad X(z) = \frac{z+2}{z(z-2)}.$$

$$(e) \quad X(z) = \frac{1 + 2z + 3z^2 + 4z^3 + 5z^4}{z^4}, \qquad (f) \quad X(z) = \frac{10z + 5}{z^2 - 1.2z + 0.2}.$$