Digital Control

CSE421

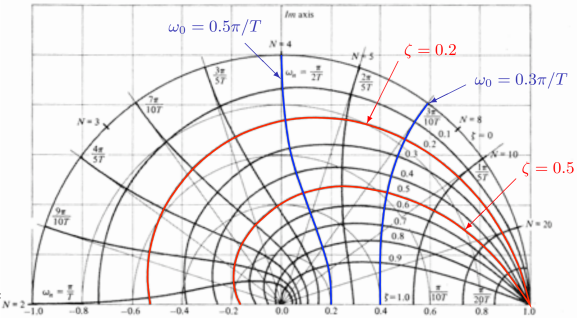
Asst. Prof. Dr.Ing.

Mohammed Nour A. Ahmed

mnahmed@eng.zu.edu.eg

https://mnourgwad.github.io

Lecture 15: **Pole placement (Servo Problem)**



Zagazig University | Faculty of Engineering | Computer and Systems Engineering Department | Zagazig, Egypt

Lecture 15:

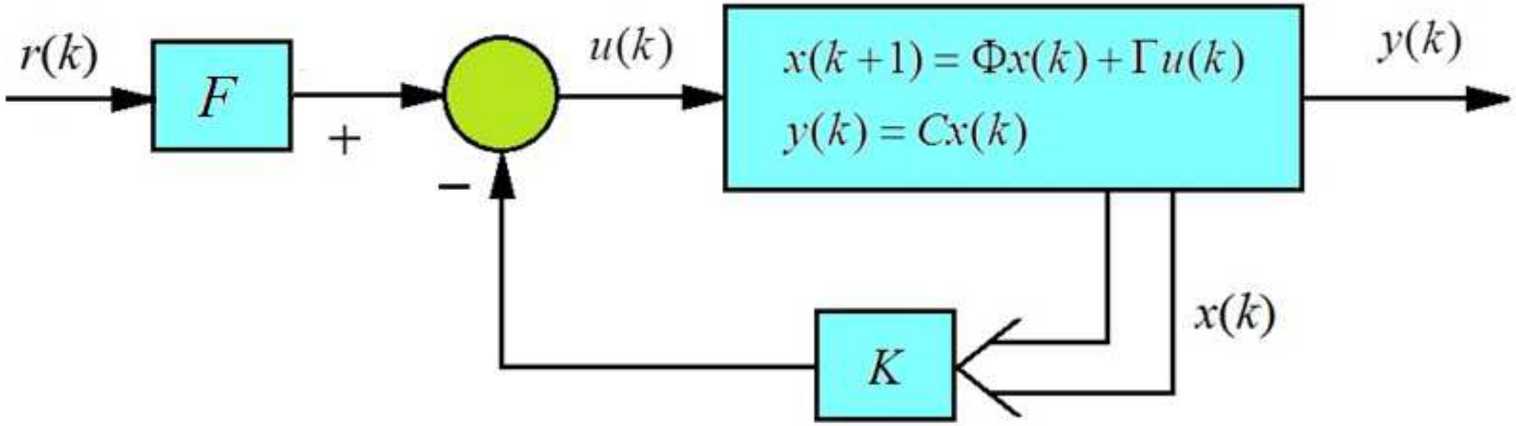
Pole placement

(Servo Problem)

Servo Problem

- The previous regulator state feedback scheme drives the system state to *zero* starting from any initial condition.

- In order to track a nonzero constant reference $\boldsymbol{r(k)}$**, we** consider the following scheme where the reference gain $\boldsymbol{F}$ insures that $\boldsymbol{y(k) = r(k)}$ at steady-state (i.e. the steady-state error is zero).



Servo Problem

Writing the system equations:

$$x(k+1) = \Phi x(k) + \Gamma u(k),$$

$$y(k) = Cx(k)$$

$$u(k) = -Kx(k) + Fr(k)$$

We can obtain:

$$x(k+1) = (\Phi - \Gamma K)x(k) + \Gamma F r(k)$$

$$y(k) = Cx(k)$$

where the closed-loop state matrix is

$$\Phi_{cl} = \Phi - \Gamma K$$

Servo Problem

- The transfer function of the whole system is:

$$G_{cl}(z) = \frac{Y(z)}{R(z)} = C(zI - \Phi_{cl})^{-1}\Gamma F$$

- In order to have zero steady-state tracking error for a unit step input, the dc gain of the system must be unity.

$$K_{dc} = G_{cl}(1) = C(I - \Phi_{cl})^{-1}\Gamma F = 1$$

- Solving for the reference gain:

$$F = \left[C(I - \Phi_{cl})^{-1}\Gamma\right]^{-1}$$

Example

In the previous lecture, we designed a state feedback controller to assign the eigenvalues {0.3$\pm$ j0.2} to the discrete-time state space system having the following matrices.

$$\Phi = \begin{pmatrix} 0 & 1 \\ 3 & 4 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

Calculate the reference gain so that the system can track step inputs with zero steady-state error.

Solution

- We will repeat the design steps for the regulator problem, then calculate the reference gain.
- For the given eigenvalues, the desired characteristic polynomial is

$$\Delta = (\lambda - 0.3 + j0.2)(\lambda - 0.3 - j0.2) = \lambda^2 - 0.6\lambda + 0.13$$

- The closed-loop state matrix is

$$\Phi_{cl} = \Phi - \Gamma K = \begin{pmatrix} 0 & 1 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} k_1 & k_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 3 - k_1 & 4 - k_2 \end{pmatrix}$$

- The closed-loop characteristic polynomial is

$$|\lambda I - A_{cl}| = \begin{vmatrix} \lambda & -1 \\ -(3 - k_1) & \lambda - (4 - k_2) \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - (4 - k_2)\lambda - (3 - k_1) = 0$$

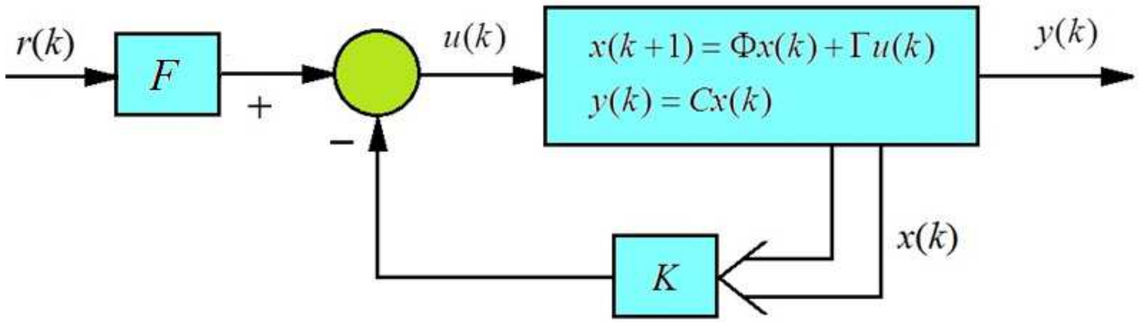
Solution

- Comparing the closed-loop polynomial with the desired characteristic polynomial gives

$$k_1 = 3.13, \quad k_2 = 3.4$$

- The closed-loop system matrix is

$$\Phi_{cl} = \begin{pmatrix} 0 & 1 \\ -0.13 & 0.6 \end{pmatrix}$$



- The feedforward reference gain is

$$F = \left[C(I - \Phi_{cl})^{-1}\Gamma\right]^{-1} = \left[\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 0.13 & 0.4 \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]^{-1} = \left[\frac{1}{0.53}\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 0.4 & 1 \\ -0.13 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]^{-1}$$

$$= 0.53$$

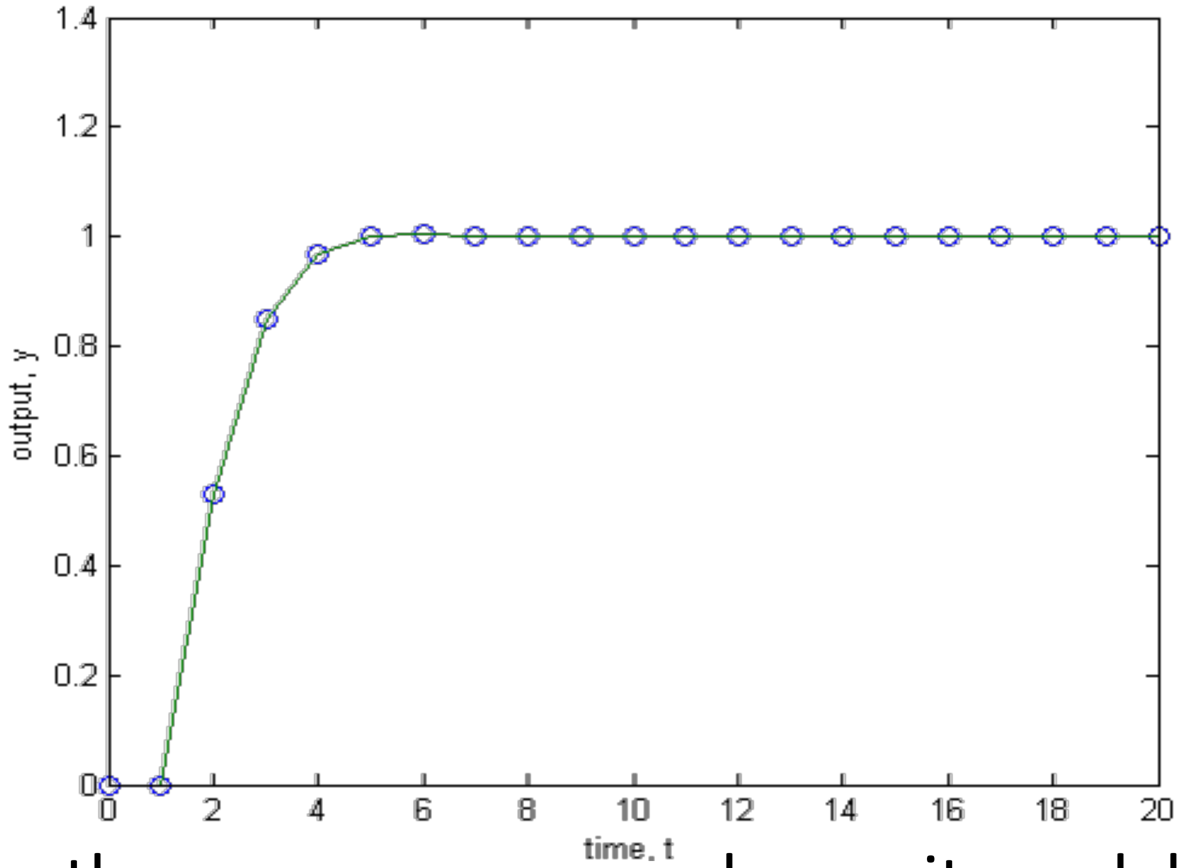
Simulation

The following m-file simulates the response of designed system to a unit-step input.

```
% The open-loop system matrices
phi = [0 1;3 4];
gam = [0;1];
C=[1 0];
% The designed feedback and feedforward gains
F = 0.53; K = [3.13 3.4];
% The closed-loop system matrices
phicl = phi-gam*K;
sysd=ss(phicl,gam*F,C,0,1);% state space model, assume sampling period T=1
[y,t]=step(sysd);              % simulate the step response
plot(t,y,'o',t,y)              % plot the step response
xlabel('time, t')
ylabel('output, y')
```

Simulation

- The response of the system $y(k)$ to a unit step reference $r\,(k)$:



- As can be seen, the response approaches unity and, hence, the **steady-state error is zero** as desired.

Drawback of the method

- The control scheme with reference gain is a **feedforward action** determined by the gain ***F*** to yield zero steady-state error for a constant reference input $r(k)$.

- Because the forward action does not include any form of feedback, this approach is **not robust to modeling uncertainties**. Thus, modeling errors (which always occur in practice) **will result in *nonzero* steady-state error**.

- To solve this problem, **integral control** can be used.

Thanks for your attention.

Questions?

Asst. Prof. Dr.Ing.

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Robotics Research Interest Group (zuR^2IG)
Zagazig University | Faculty of Engineering | Computer and Systems Engineering Department | Zagazig, Egypt