Digital Control

CSE421

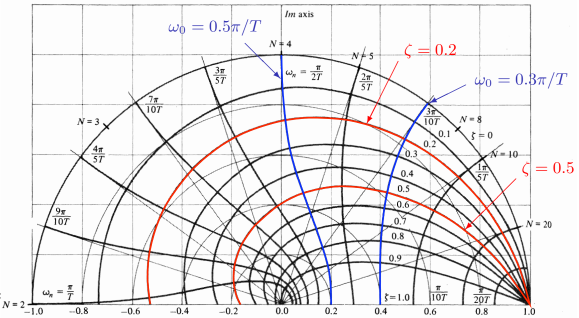
Asst. Prof. Dr.Ing.

Mohammed Nour A. Ahmed

mnahmed@eng.zu.edu.eg

https://mnourgwad.github.io

Lecture 11: **Discrete Controller Design (PID Controller)**



Zagazig University | Faculty of Engineering | Computer and Systems Engineering Department | Zagazig, Egypt

Lecture: 11

Discrete Controller Design (PID Controller)

- PID Controller

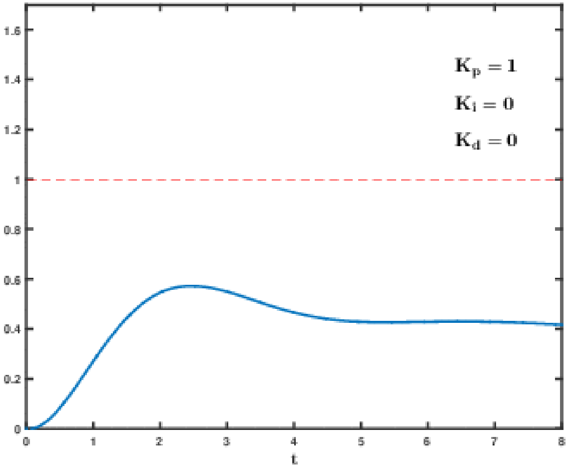
PID controller

- The proportional–integral–derivative (PID), also called three-term, is the most widely used controller in process industry.
- The output u(t) of the PID controller is the sum of three terms:

$$u(t) = K_p \left(e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right)$$

- where
  - $e(t) = r(t) - y(t)$, is the error (controller input)
  - $r(t)$ is the reference input
  - $y(t)$ is the plant output.
  - T_i is known as the integral time.
  - T_d is known as the derivative time.

PID controller



https://commons.wikimedia.org/wiki/File:PID_Compensation_Animated.gif

PID controller actions

- **Proportional**: the error is multiplied by a gain. The higher is the gain, the faster is the response. However, very high gain may cause instability. Note that system with only P-control has a steady-state error (Offset).
- **Integral**: is used to remove steady-state error. However, integral action increases the overshoot and reduces system stability.
- **Derivative**: is used to improve the transient response by reducing overshoot.

PID controller

Transfer Function

- By taking Laplace transform of the equation:

$$u(t) = K_p \left(e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right)$$

- we obtain the transfer function of the continuous-time PID controller:

$$\frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$

Discrete PID Controller

- To implement PID control using a digital computer we convert the following continuous-time equation into a discrete form:

$$u(t) = K_p \left(e(t) + \frac{1}{T_i} \int\limits_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right)$$

- To do this, a simple method is to approximate integral and derivative using finite differences:

$$\int\limits_0^t e(\tau) d\tau \approx \sum_{k=1}^{n} T\, e(kT),$$

$$\frac{de(t)}{dt} \approx \frac{e(nT) - e(nT - T)}{T}$$

Discrete PID controller

position form

- Using finite difference approximations, we can write:

$$u(nT) = K_p \left[e(nT) + \frac{1}{T_i} \sum_{k=1}^{n} Te(kT) + T_d \frac{e(nT) - e(nT - T)}{T} \right]$$

- Using subscripts instead of arguments, then

$$u_n = K_p \left[e_n + \frac{1}{T_i} \sum_{k=1}^{n} T e_k + T_d \frac{e_n - e_{n-1}}{T} \right]$$

- This is called the **position form** of discrete PID controller. The drawback of this form is that: to calculate the controller output u_n we need error values e_k, $k = 1 \rightarrow n$.

Discrete PID controller

velocity form

- From the position form:

$$u_n = K_p\left[e_n + \frac{1}{T_i}\sum_{k=1}^{n} T e_k + T_d\frac{e_n - e_{n-1}}{T}\right]$$

- We can write

$$u_{n-1} = K_p\left[e_{n-1} + \frac{1}{T_i}\sum_{k=1}^{n-1} T e_k + T_d\frac{e_{n-1} - e_{n-2}}{T}\right]$$

- Subtracting these two equations, we obtain:

$$u_n = u_{n-1} + K_p[e_n - e_{n-1}] + \frac{K_p T}{T_i}e_n + \frac{K_p T_d}{T}[e_n - 2e_{n-1} + e_{n-2}]$$

- Here the current control signal u_n is an update of the previous value u_{n-1}. This is called the **velocity form**.

Transfer function of Discrete PID controller

- The velocity form of discrete PID controller is:

$$u_n = u_{n-1} + K_p[e_n - e_{n-1}] + \frac{K_p T}{T_i} e_n + \frac{K_p T_d}{T}[e_n - 2e_{n-1} + e_{n-2}],$$

$$u_n - u_{n-1} = \underbrace{K_p\left(1 + \frac{T}{T_i} + \frac{T_d}{T}\right)}_{K_0} e_n + \underbrace{K_p\left(-1 - 2\frac{T_d}{T}\right)}_{K_1} e_{n-1} + \underbrace{K_p\left(\frac{T_d}{T}\right)}_{K_2} e_{n-2}.$$

- Taking z-transform of both sides, we get the transfer function of discrete PID controller:

$$\frac{U(z)}{E(z)} = \frac{K_0 + K_1 z^{-1} + K_2 z^{-2}}{1 - z^{-1}}$$

Transfer function of Discrete PID controller

- Transfer function of discrete PID controller is:

$$\frac{U(z)}{E(z)} = \frac{K_0 + K_1 z^{-1} + K_2 z^{-2}}{1 - z^{-1}}$$

where:

$$k_0 = K_p \left(1 + \frac{T}{T_i} + \frac{T_d}{T}\right)$$

$$k_1 = -K_p \left(1 + 2\frac{T_d}{T}\right)$$

$$k_2 = K_p \left(\frac{T_d}{T}\right)$$

Thanks for your attention.

Questions?

Asst. Prof. Dr.Ing.

Mohammed Nour A. Ahmed

mnahmed@eng.zu.edu.eg

https://mnourgwad.github.io

ZU Robotics Research Interest Group (zuR^{2}IG)

Robotics Research Interest Group (zuR^{2}IG)
Zagazig University | Faculty of Engineering | Computer and Systems Engineering Department | Zagazig, Egypt