Digital Control

CSE421

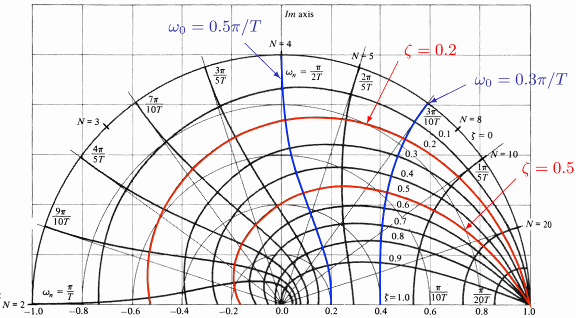
Asst. Prof. Dr.Ing.

Mohammed Nour A. Ahmed

mnahmed@eng.zu.edu.eg

https://mnourgwad.github.io

Lecture 10: **Discrete Controller Design (Deadbeat & Dahlin Controllers)**



Lecture: 10

Discrete Controller Design (Deadbeat & Dahlin Controllers)

- Deadbeat controller
- Dahlin controller

Deadbeat Controller

- Its aim is to **bring the output to steady state in smallest number of time steps**
  - assuming, for simplicity, that the set point is a step input.

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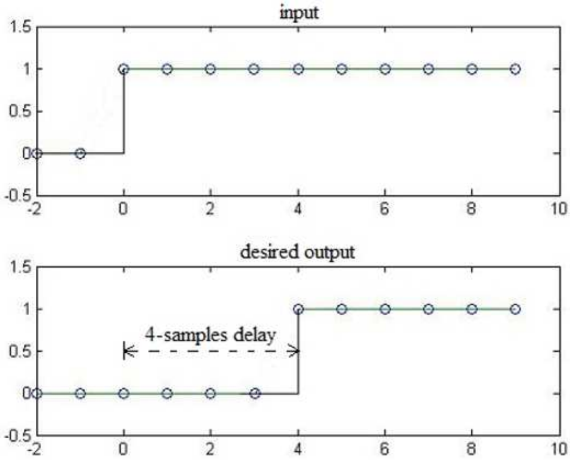

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- It is interesting to note that deadbeat control is equivalent to placing all closed-loop poles at $z = 0$.
- These poles correspond to the fastest response possible.
  - Usually such requirement will come at the expense of **large control signal**.

Deadbeat Controller

Example

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The open-loop transfer function of a plant is given by:

$$G(s) = \frac{e^{-2s}}{10s + 1}$$

Design a dead-beat digital controller for the system. Assume that T = 1 s.

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Design a dead-beat digital controller for the system. Assume that T = 1 s.

- The transfer function of the system with a ZOH is given by

$$GH(z) = \mathscr{Z}\left\{\frac{1-e^{-Ts}}{s}G(s)\right\} = (1-z^{-1})\mathscr{Z}\left\{\frac{e^{-2s}}{s(10s+1)}\right\}$$

$$= (1-z^{-1})z^{-2}\mathscr{Z}\left\{\frac{1}{s(10s+1)}\right\}$$

- From the z-transform tables

$$\mathscr{Z}\left\{\frac{a}{s(s+a)}\right\} = \frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$$

$$\text{So,}\qquad GH(z) = (1-z^{-1})z^{-2}Z\left\{\frac{0.1}{s(s+0.1)}\right\}$$

$$= (1-z^{-1})z^{-2}\frac{z(1-e^{-0.1})}{(z-1)(z-e^{-0.1})}$$

$$= \frac{0.095}{z^3-0.904z^2}$$

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- Hence, the dead-beat controller is given by:

$$D(z) = \frac{1}{GH(z)}\frac{T(z)}{1-T(z)} = \frac{z^3-0.904z^2}{0.095}\left(\frac{1}{z^k-1}\right)$$

Deadbeat Controller

Example

- For realizability, we must choose $k \geq 3$.

Deadbeat Controller

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- Choosing $k = 3$, we obtain the controller:

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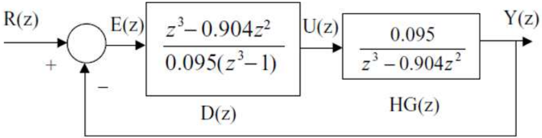
- With this controller, the block diagram of the closed-loop is:



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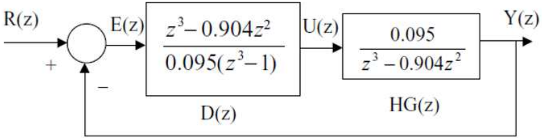


- To analyze the designed system performance, we simulate the closed-loop step response and the control signal.

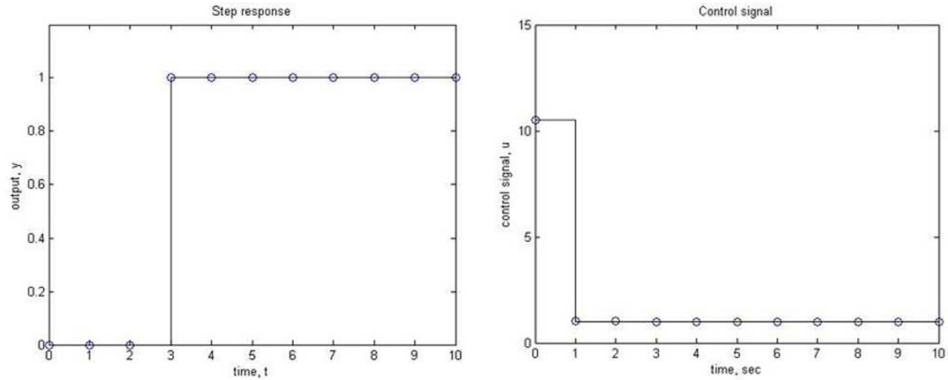
```
% Deadbeat control: D(z) = (z^3 - 0.904 z^2) / (0.095 (z^3 - 1))

Gp = tf(1,[10 1],'iodelay',2);
Gpd = c2d(Gp,1);
Gc = tf([1 -0.904 0 0],[0.095 0 0 -0.095],1);
Gcl=Gc*Gpd/(1+Gc*Gpd);
t=0:1:10;
y=step(Gcl,t)
figure; plot(t,y,'o'); hold on; stairs(t,y); hold off
xlabel('time, t'), ylabel('output, y'), axis([0 10 0 1.2]),title('Step response')

Gru=Gc/(1+Gc*Gpd);
u=step(Gru,t)
figure; plot(t,u,'o'); hold on; stairs(t,u); hold off
xlabel('time, sec'), ylabel('control signal, u'), axis([0 10 0 15]), title('Control signal')
```

Deadbeat Controller

Example



- As desired, the step response is unity after 3 seconds.

Deadbeat Controller

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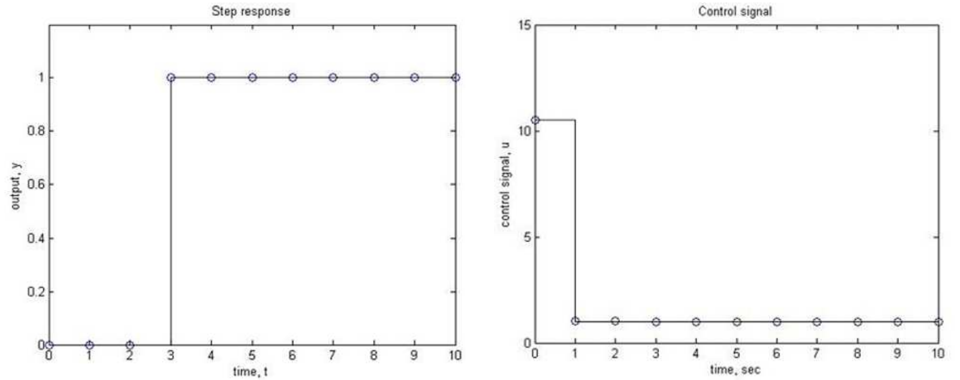
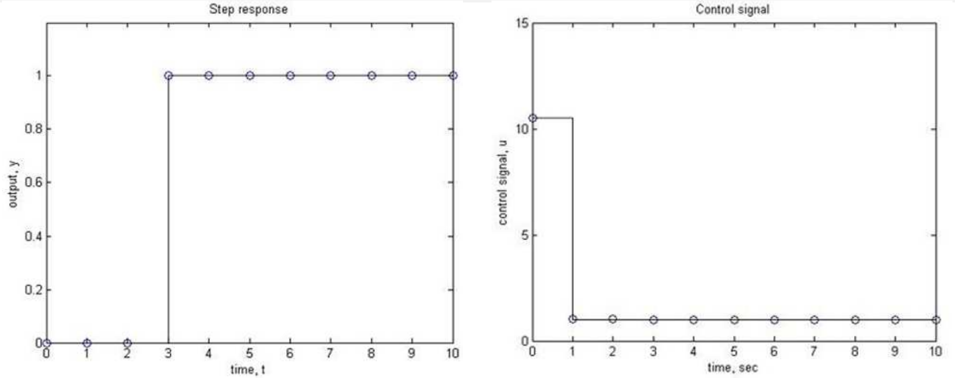


- As desired, the step response is unity after 3 seconds.
- It is, however, important to realize that the response is correct only at the sampling instants and the response can have an oscillatory behavior between samples.

Deadbeat Controller

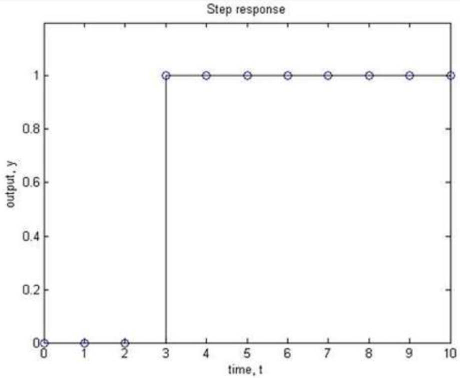
Example

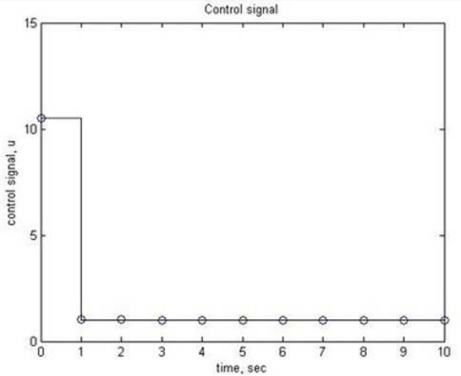


- We realize that the magnitude of the control signal is very large at the beginning (≈ 11).

Deadbeat Controller

Example





- We realize that the magnitude of the control signal is very large at the beginning (≈ 11).

The main drawback of dead-beat control is that it requires excessive (large) control efforts which may not be acceptable in practice.

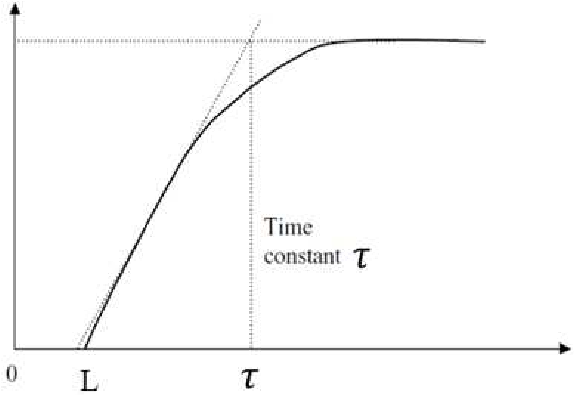
Dahlin Controller

- Dahlin[1] controller is a **modification of the deadbeat controller** which produces an exponential response that is smoother than deadbeat response.

[1] Eric Dahlin worked for IBM in San Jose then for Measurex in Cupertino.

Dahlin Controller

- Dahlin[1] controller is a **modification of the deadbeat controller** which produces an exponential response that is smoother than deadbeat response.
- The desired closed-loop response for step input looks like:



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Dahlin Controller

- Hence, the desired closed-loop transfer function is:

$$G_{cl}(s) = \frac{e^{-Ls}}{\tau s + 1}$$

Dahlin Controller

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- As step input is assumed (which is constant between samples), the desired closed-loop transfer function in the z-domain will be:

$$T(z) = \mathscr{Z}\{G_{zoh}(s)\ G_{cl}(s)\} = \mathscr{Z}\left\{\frac{1 - e^{-Ts}}{s}\frac{e^{-Ls}}{\tau s + 1}\right\}$$

Example

The open-loop transfer function of a plant is given by:

$$G(s) = \frac{e^{-2s}}{10s + 1}$$

Design a Dahlin digital controller for the system to achieve a closed-loop time constant of 5 s. Assume that $T = 1$ s.

Dahlin Controller
Example

- First, we need to find the z-transform of the process (preceded by a ZOH). From the previous example, this is found to be:

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$$GH(z) = \frac{0.095}{z^3 - 0.904z^2}$$

- Second, we need to find the z-transform of the desired closed-loop transfer function, $T(z)$.
- As the desired closed-loop time constant, τ, is 5 sec,

$$T(s) = \frac{e^{-Ls}}{5s + 1}$$

Dahlin Controller

Example

- Therefore,

$$\begin{aligned} T(z) &= \mathscr{Z}\left\{ \frac{1-e^{-sT}}{s} \frac{e^{-Ls}}{5s+1} \right\} \\ &= (1-z^{-1})z^{-L/T}\,\mathscr{Z}\left\{ \frac{1}{s(5s+1)} \right\} \\ &= (1-z^{-1})z^{-k}\,\mathscr{Z}\left\{ \frac{0.2}{s(s+0.2)} \right\} \\ &= (1-z^{-1})z^{-k}\frac{z(1-e^{-0.2T})}{(z-1)(z-e^{-0.2T})} \\ &= z^{-k}\frac{(0.181)}{(z-0.819)} \end{aligned}$$

Dahlin Controller

Example

- Therefore,

$$T(z) = \mathscr{Z}\left\{\frac{1-e^{-sT}}{s}\frac{e^{-Ls}}{5s+1}\right\}$$
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$$= z^{-k}\frac{(0.181)}{(z-0.819)}$$

- The Dahlin controller is thus given by:

$$D(z) = \frac{1}{G(z)}\frac{T(z)}{1-T(z)}$$
$$= \frac{z^3-0.904z^2}{0.095}\frac{z^{-k}\frac{(0.181)}{(z-0.819)}}{\left(1-z^{-k}\frac{(0.181)}{(z-0.819)}\right)}$$
$$= \frac{z^3-0.904z^2}{0.095}\frac{0.181z^{-k}}{z-0.819-0.181z^{-k}}$$
$$= \frac{0.181z^{3-k}-0.164z^{2-k}}{0.095z-0.078-0.017z^{-k}}.$$

Dahlin Controller

Example

- For the controller to be **realizable**: degree of numerator must be $\leq$ degree of denominator

$$3 - k \leq 1 \quad \Rightarrow \quad k \geq 2$$

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- Using the designed controller, the closed-loop step response and control signal are simulated next.

Dahlin Controller

MATLAB code for Example

```
Gp = tf(1,[10 1],'iodelay',2);
Gpd = c2d(Gp,1);
Gc = tf([0.181 -0.164 0 0],[0.095 -0.078 0 -0.017],1);
Gcl=Gc*Gpd/(1+Gc*Gpd);
t=0:1:30;
y=step(Gcl,t);

figure; plot(t,y,'o',t,y)
xlabel('time, t'), ylabel('output, y')
axis([0 30 0 1.2])
title('Step response')

Gru=Gc/(1+Gc*Gpd);
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figure; plot(t,u,'o'); hold on; stairs(t,u); hold off;
xlabel('time, sec'), ylabel('control signal, u')
axis([0 30 0 5])
title('Control signal')
```
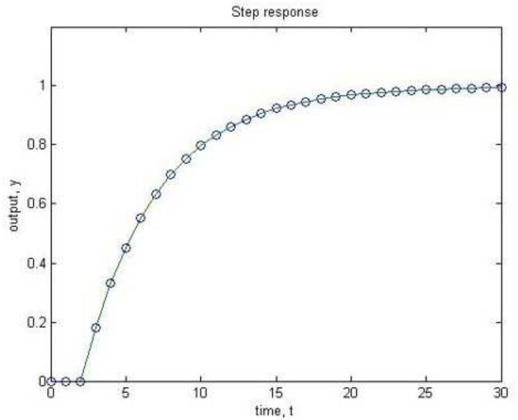
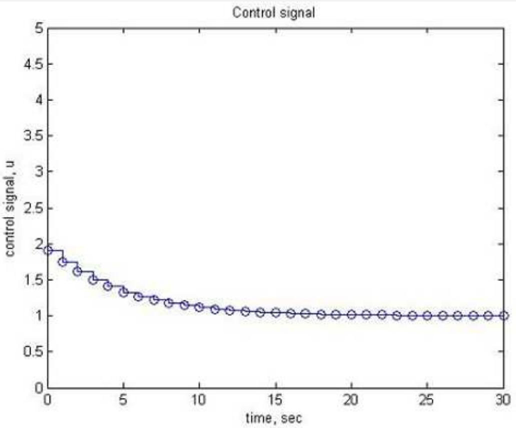

Dahlin Controller

Example step response





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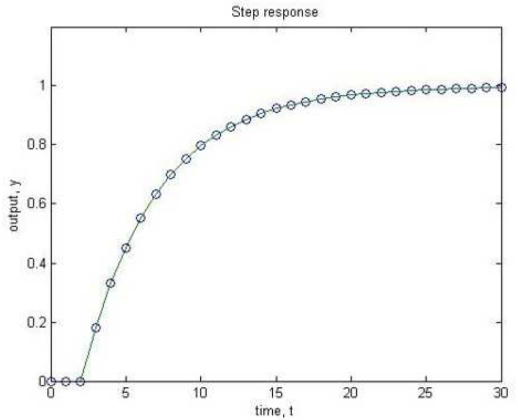

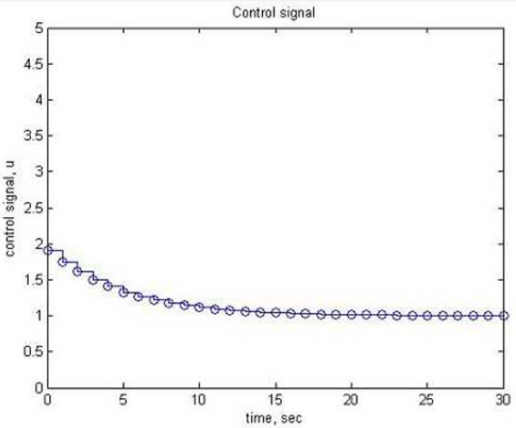

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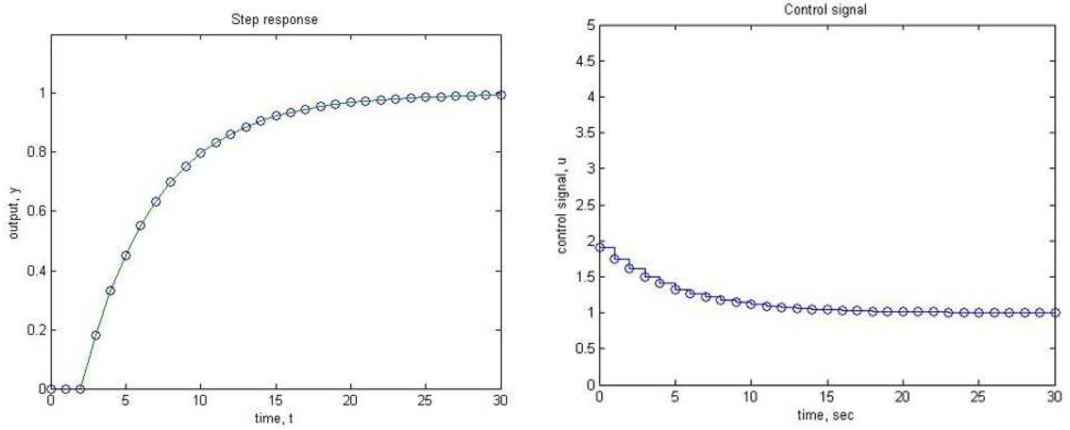
- What is the time delay? time constant?

Dahlin Controller

Example step response



- the maximum control signal magnitude (≈ 1.9) is much smaller than the control signal obtained using a deadbeat controller (≈ 11). This is more acceptable in practice.

Thanks for your attention.

Questions?

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Robotics Research Interest Group (zuR^{2}IG)
Zagazig University | Faculty of Engineering | Computer and Systems Engineering Department | Zagazig, Egypt